

“Vagueness” is Context-Dependence. A Solution to the Sorites Paradox

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It is argued in this paper that the vagueness of natural language predicates arises from the fact that they are learned and used always in limited contexts and hence are incompletely defined. A semantics for natural language must take this into account by making the interpretation of predicates context-dependent. It is shown that a context dependent semantics also provides the means for an account of vagueness. These notions are first developed and argued for in abstract terms and are then applied to a solution of the prototype of vagueness puzzles: the paradox of the heap.

0. Introduction

For all we know, it was Eubulides the Megarian who invented the well-known paradox of the heap, which may be put as follows¹: Suppose there is a heap of grain. And suppose that it is true of any heap of grain that it remains a heap after one grain has been removed. If we grant these two premisses, however, we will also have to grant that there is still a heap of grain when, after all grains have been removed one by one, not a single grain is left.

This conclusion is plainly unacceptable though both of the premisses would seem to be true and also the rule of *modus ponens* would seem beyond doubt. A natural reaction in this situation is to put the blame on the vague predicate “is a heap”. But if this reaction in turn leads to the conclusion that formal reasoning in the presence of vague predicates would have to be abandoned, it will be hard to settle for it. The most popular resolution that has been propagated for this unhappy situation is the proposal of “logistics of vagueness”.

In this paper, we do not want to discuss any quarrels one might have with logics of vagueness. Instead, we want to turn to a number of questions which vagueness-logics have turned away from and which are in danger of being obscured: we want to gain some conceptual clarity about what vagueness in natural language is, how it arises, and why it should cause trouble for formal reasoning. As it happens, one of the results of these considerations will be that vagueness-logics are theoretically superfluous for natural language semantics. Vagueness, as we shall see, is a straightforward consequence of context-dependence of interpretation and can be accounted for by the same means that account for the context-dependence of interpretation in natural language semantics. Nonetheless, as we shall see, there remains an important practical role for vagueness-logics.

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1. The Root of Vagueness is Incomplete Definition

We may help ourselves to the essential ingredients for a treatment of vagueness from the semantics of Gottlob Frege.

The sort of things he attributed impreciseness to are concepts. And concepts, for Frege, are functions from things into the truth-values. Concepts are also the referents (*Bedeutungen*, as Frege has it) of predicate-expressions. Now, a concept is imprecise or (as we shall prefer) vague, if and only if it is not defined for each and every argument.

When Frege proposed this notion of vagueness (1903: 69f.), he was concerned with arithmetic. But he clearly had a more general conception of vagueness in mind. He wrote, for instance:

“Does the question ‘Are we still Christians?’ really make sense if it is not clearly determined of which individuals the predicate *Christian* may truthfully be asserted and from which individuals it must be withheld?” (l.c.)

But Frege not only provides us with a clear notion of vagueness, he also gives us a good idea as to the origin of the disease.

He complains about the regrettable habit of mathematicians who define their concepts piecemeal, e.g. by first defining a particular concept for a special case, say for the domain of positive integers, and by then employing the concept for various theorems until, at a convenient moment, they decide to add a second definition, perhaps for the number zero and the negative integers, etc.

Frege’s objections to these ad hoc definitions are that (i) there is a great risk of overlap between different definitions of one concept that are given in different contexts. Contradictions thus cannot, in principle, be excluded. (ii) A disorderly lot of such definitions does not ordinarily add up to a complete definition of the concept in question. More likely, there are going to be gaps with arguments for which the concept has accidentally remained undefined; still, Frege continues, “some people are naive enough to use the word or sign also in these cases [which have been left out in the definitions] as if they had given it a meaning.” (1903: 70).

Again, we are not in the first instance worried about the actual case of arithmetical concepts, with which Frege is concerned. What strikes us is rather how neatly Frege’s description of the situation in mathematics captures central properties of the processes by which we acquire the use of the expressions in our mother tongue, or, if you wish, the concepts underlying them.

Also in the acquisition of our native language we get started with partial explanations of the use of words; in the extreme case, these partial explanations are ostensive definitions². We are shown positive and negative instances and we are told: “This is a rose, and also that and that; but this here is not a rose.” But in the majority of cases we are probably not even given any explicit definition at all, not even an ostensive one. Rather, we have to pick up the use of an expression from the various accidental concrete applications we happen to come across. Now, these are always applications to a limited number of things, always in different contexts. In response, we build up, as rationalists would probably have it, a system of definitions that can form the basis for our own use of the expressions. But Frege’s considerations would induce a more empiricist outlook, and we are more inclined to picture the language-learning-situation

² The theoretical consequences of this fact for a semantic theory of natural language can hardly be overestimated. And the very fundamental revision of the Frege-Carnap tradition in semantics that is called for in order to account for ostension concepts as the core of natural language is a task that lies still ahead.

as something rather less systematic. Bit by bit, we might say, the child who is exposed to those partial explanations witnesses the growth of something more appropriately called a jungle of definitions: intersecting, contradicting each other, complementing each other, pushing each other aside, and still leaving whole areas of untouched desert land in between and wide unexplored oceans around the buzzing jungle. Still, as long as we move in our jungle, somehow we seem to be doing alright, communication flows uninhibited. Only when we find ourselves unexpectedly in one of the deserts or are driven out into the ocean, we stand speechless: for those regions our predicates are not defined. Take for instance the notorious table that used to haunt Oxford discussions: a table that stands right in front of you now and that is gone when you open your eyes again after you had closed them for a second and that is back where it first stood after another blink, etc. Do we want to call that thing (or whatever it is) a “table”? – Or take Putnam’s example: suppose, we discover that the animals we have known and loved as cats are actually robots, remote-controlled from another star. Do they still fall under our concept of cats?

Waismann (1968) speaks of “open texture” of concepts where it comes to outlandish cases like these. And he wants to distinguish open texture from vagueness by suggesting that predicates like “is a heap”, as in the *Sorites*, are *actually vague* whereas open texture rather is the *possibility of vagueness*. The intuition seems clear: everybody who knows the word “heap” knows that there is no clear borderline between a heap and something that is just too little to be a heap. But, on the other hand, until we are confronted with Putnam’s exotic case, we seem to delude ourselves that we do know the difference between a cat and a robot. As it happens, we are confronted with heaps and non-heaps of all sizes fairly frequently, but we are not so often bothered by cases like Putnam’s.

Still, we find this distinction rather too accidental in order to accept it as the basis for a conceptual distinction, and we would insist that also open texture should fall under the notion of vagueness: in each of the cases just given, the concept lacks a specification with respect to certain properties of the things it is to be applied to or with respect to situation in which it is applied to those things.

But is there not another difference between vagueness and open texture that has to do with the mere limitation of our senses? Is not the concept of a heap vague, just because the transition from a “proper” heap to what is no longer a heap may proceed in steps too small for us to perceive? And should not rather this lack of discriminability be made the basis of vagueness, whereas incomplete definition would rather be what is at the roots of open texture?

Now this suggestion would seem to imply that concepts that are tied up immediately with sense-experience are vague by necessity, whereas concepts that are not directly linked to sense-experience are not vague in ordinary use but may have an open texture.

Michael Dummett (1975) has argued, however, that there can be concepts, immediately linked to sense-experience, that are still not vague. We might construe, for instance, a *comparative concept* of something being blue by saying that anything is to count as blue that is not discriminably different from the colour given by a particular sample from a particular colour chart. Either, one should think, we can discriminate the colour of a particular surface from the one on our chart, or we cannot. There is no third possibility. – Certainly, there are problems about the notion of discriminability: discriminable by whom? With the naked eye or with auxiliary devices? And if without auxiliary devices: are glasses auxiliary devices? If they are, how about the man who is almost blind without them? If glasses are not auxiliary devices, where is the borderline to a magnifying glass or a microscope? – Perhaps these questions are not so essential for the colour-predicate, but they are essential for a general notion of

discriminability as it is needed for this type of comparative definition³. Still, we may solve this problem in one or the other pragmatic fashion. Let us suppose we have already done that, in order to take a look at another difficulty that will then arise immediately and that is more important in our present context of discussion. Again, it may only seem a pragmatic problem that, in order to have even quite a small language of comparative predicates and to be able to talk about only very simple things in very simple situations, we would have to carry truckloads of samples with us. But there is more than a pragmatic problem in carrying a copy of the universe with us in order to be able to talk about anything we please. All our concepts would be precise, but we would have to have infinitely many. In fact, we would have to be omniscient: on the basis of complete knowledge of everything experience may have to offer, i.e. on the basis of possessing a complete copy of the universe, we could have precise concepts – even concepts that immediately link up to sense experience.

Hence we take it that it is not any limitation of our senses which lies at the roots of vagueness, nor anything that would be specific to experience. Here, as elsewhere, the common problem is that we do not possess complete knowledge of the domain of arguments for which our concepts are to be defined, and thus we cannot provide a complete definition of our concepts. Dummett's comparative concepts show that we have a choice, however, between either having incompletely defined concepts or having all and only completely defined concepts but being unable to define a sufficient number of them in order to cover the infinite domain of things we may need our concepts for.

2. Precise Communication by Means of Vague Terms

The account of vagueness just sketched, which identifies vagueness with incomplete definition, has the considerable advantage over its rivals of being able to accommodate in a straightforward fashion the fact that precise communication is possible, even by means of “vague” ordinary language. We would thus be relieved from the hypocrisy of having to claim professionally that natural language is inevitably vague, while in every day life we proceed as if all was well and simply rely on precise communication.

How then can we account for precise communication? We said earlier that concepts should be regarded as the referents of predicate-expressions. Let us now add that the reference of predicate-expressions is to be seen as depending on the context; i.e. predicate-expressions are functions from contexts to concepts. This view should be plausible already in its own right, independent of any facts about vagueness. When I say that the traffic lights are red I am certainly not using the same concept of being red as in a situation where I say that a tomato is red. A red grapefruit won't have the same colour as a red tomato (in fact, it will be yellow all over), and red salmon still won't have the colour-distribution of the traffic lights or the grapefruit or the tomato⁴.

Let us further assume that for each concept there is at least one context for which it is completely defined. One such context would ordinarily be the context in which the concept has first been acquired. It would certainly be a poor joke to attempt to teach someone how to use the word “chair” under circumstances like those in Max Black's imaginary museum (where there is a row of thousands of objects, a beautiful museum piece of a chair at one end and a rough block of wood at the other, and where there is no visible difference between each

³ Cf. the discussion of the notion of discriminability in Goodman (1976).

⁴ Some of the examples are borrowed from Travis (1976); cf. Bosch (1979) for further discussion.

two neighbouring objects). Nor would anybody try to introduce a child to the notion of a heap in view of a situation where there are objects ranging from mountains to single grains of sand.

But apart from actual introductory contexts there are certainly more contexts for each concept where it is completely defined, i.e. where for each object in the context there is complete clarity as to whether it falls under the concept in question or not. To assume this is only reasonable, since there are plainly many contexts in which a particular concept *could* have been acquired by a particular person, next to the context in which that person *actually* did acquire the concept. Also, there are infinitely many contexts that are identical in all relevant respects to any particular learning context but that differ from the learning context in other respects, irrelevant for the concept at issue.

In fact, what matters for the completeness of definition is not the actual *context in rebus* but the way the speakers and listeners see the context: i.e. their mental model of the context in rebus, their context-model. Objects that are in fact in the actual context but that are not noticed or are not paid attention to by speaker and listener, i.e. that are not in their context-models, never raise the question whether or not a particular concept applies to them.

The next assumption we have to make concerns the notion of understanding. Ordinarily, when people say something, they mean to say something in particular. Hence they must be employing context-models in respect of which the predicates they are using yield concepts that are completely defined in those context-models. For what could someone possibly mean to say who utters the sentence “May I sit on this chair?”, without indicating any particular object, in view of a situation as in Max Black’s museum? In any ordinary kind of context-model we can imagine for that sort of situation, there would not be one unique object to qualify as a chair.

Or, suppose, I ask someone to get a book from my desk, which I left there, and I describe the book as red. If he comes back with three books in various shades of red, plainly, something has gone wrong. It may be that I employed a context-model according to which there was only one of these books on my desk, so that the description “red book” would have been sufficient, but that, in fact, there were also these other books, which I had forgotten about. Then the listener could not, in view of the books that actually were there, reconstruct my context-model (in respect of which the predicate “red book” was completely defined), but was forced to construct a different model in respect of which that predicate was not completely defined. Alternatively, the listener may have gone to the wrong office and may have taken someone else’s books. Again: we find a mismatch of context-models.

Hence, ordinarily, when we find that someone said something vague, we may well not have found the right context-model, that is the one his utterance was assuming. Our context-model would then differ from his in that there are objects represented in our model that are not in his and with respect to which the relevant concept turns out to be vague. Those objects simply cannot have been in the speaker’s model, if he was intending to say something in particular. For there is no particular thing, i.e. no completely defined concept, his utterance would have referred to if those objects had been in his context-model.

However, precise communication is not merely a matter of the listener’s effort. It is an interactive process and also demands that the speaker adjusts to his audience. If he knows sufficiently well what he is talking about, i.e. if he has a fairly appropriate model of the context at issue, and if he has also a good idea of how his audience would ordinarily model that context, then he must be able to select expressions that would yield completely defined concepts with respect to the context as the audience will reconstruct it. He may, for instance, add specifications to his predicates in order to bar the listener from selecting particular unintended concepts that might be a likely choice in view of the situation if the specification

is absent. In some places one must add, when ordering a whisky: “but no ice please” in order to prevent the barman from the barbarism of selecting a concept of whisky according to which the drink is served with ice. And in some places one has to add this specification even when one orders a brandy. In other places, the barman might wonder what odd assumptions I make about him (or his context-model) if I specify “no ice please”⁵.

Note that the above description of vagueness arising in ordinary communication and the corresponding description of how precise communication is possible strictly rest on the assumption of concepts being defined for the limited domain of a particular model at issue. According to the currently still wide-spread view that; concepts should be defined universally and once-and-for-all, precise communication must remain a complete mystery and there would have to be vagueness everywhere. Such is the price one has to pay for complete generality and aprioricity. On the view we are here putting forward, however, understanding each other precisely is due to an interactive effort of speaker and audience, and is always limited to particular contexts, small enough to be within the scope of the speaker’s and hearer’s awareness. Human communication, in general, according to the view here defended, is a matter of reconstructing the other’s context-model as a part of one’s own context-model (“taking the role of the other”, as George Herbert Mead had it)⁶.

To sum up: we take it that ordinarily, that is when the speaker has something definite in mind he wants to say, the concepts he employs are completely defined with respect to his own context model at that moment. If the listener finds that a particular concept, apparently employed by the speaker, turns out vague, then he has not yet found a correct reconstruction of the relevant parts of the speaker’s context model. When speaker and listener have all relevant parts of their context models in common, a fairly ordinary situation in everyday communication, then completely precise communication is possible: the speaker will quite automatically employ concepts that are completely defined with respect to the common context-model. Another matter, however, is the question whether a concept turns out vague with respect to a context the speaker did not foresee, i.e. a context that is not adequately modelled by his context-model. This, we should think, is hardly a matter of precision or vagueness of communication, but rather a matter of being able to predict what might or might not be the case.

3. Vagueness Reconstructed as Context-Dependence

After the above considerations it would not be very reasonable to insist that concepts, i.e. functions from things into the truth-values, are ordinarily, as they are used in natural language communication, incompletely defined or vague. But since we have explained vagueness, following Frege, as resting on incomplete definition, and since we also have another kind of function, namely predicate-expressions, it might be rather inviting now to swap the claim of vagueness of concepts for the claim that it is predicate-expressions, rather than concepts, that are vague. For somehow, the notion that communication in natural language is vague would seem not to be entirely unfounded.

Predicate-expressions would then be incompletely defined in the sense that a predicate does not yield a concept as its value for each and every context. And this claim is indeed correct. Only, one may wonder if this notion of vagueness is not really superfluous. For, in fact, it is nothing over and above ordinary. context-dependence of interpretation.

⁵ Cf. Searle (1978) for discussion of similar examples.

⁶ This notion of understanding was developed in Bosch (1973).

One might object though that *context-dependence of the interpretation* of predicates, which one may want to grant without further question, still does not imply the *lack of interpretation* for some contexts, i.e. does not imply incomplete definition or “vagueness” .

In abstract this point is correct, but if there is to be a difference between context-dependence of interpretation and lack of interpretation for some contexts, we must have a way of defining functions from contexts to interpretations (i.e. in this case, to concepts) completely, that is for all contexts. For any such definition, however, we would have to be able, first of all, to construe the set of all contexts. Although we are quite able to understand what one context is, or to understand the notion of any finite set of contexts, we may still be unable to understand the notion of the set of all contexts.

There would be no problem here, if we could assume the set of contexts to be a finite set. In a finite set we can distinguish the elements in any of a number of ways. But for infinite sets we need an algorithm in order to identify the set, i.e. a general criterion that distinguishes any two elements. And such a criterion, we want to argue, cannot exist for contexts. In actual fact, every two contexts are different; hence, if we decide in our reconstruction of contexts that two contexts, say c_1 and c_2 should count as identical, i.e. should be represented by the same context-model, this is always a decision which must be justified in terms of particular purposes, i.e. in terms of one particular . context. Thus we can talk about the identity of two contexts, c_1 and c_2 with respect to a third context, c_3 . But this does not tell us, how to distinguish c_3 from other contexts. Clearly, we can repeat the procedure of context-relative identification of contexts also on the next higher level and so on *ad nauseam*. But all we ever get is a context-relative notion of identity of contexts and not an absolute criterion for the identity of contexts. Accordingly, we cannot construe the notion of *the* set of all contexts.

Note that it is of no help to swap contexts in this argument for context-models. Even if we can construe the set of all context-models and define our expressions as functions from context-models to interpretations, our expressions will still not be completely defined for all experience may have to offer, because, eventually, context-models are themselves concepts of contexts, i.e. functions from contexts to truth-values. And then we have to face the same problem of construing the set of contexts on this level.

We have seen then, in this section and the preceding one, that it is not a reasonable assumption in view of the functioning of . natural language in communication that concepts are vague; and we have seen that the vagueness of predicate-expressions is no more than an accompanying feature of the context-dependence of their interpretation, which, accordingly, is most naturally dealt with in a semantic theory of natural language by the same devices that are employed for the description of context-dependence. Any full treatment of context-dependence of interpretation in natural language will automatically be a full treatment of vagueness in natural language.

4. To Interpret an Utterance is to Change one's Context-Model⁷

Before we can turn to the resolution of the Sorites paradox, we still have to clarify some notions concerning the interpretation of utterances and their parts.

Already at the very beginning we suggested that predicate-expressions should be regarded as functions from contexts to concepts. But we have not said what mechanisms would go into these functions and how they are to be computed. How does a listener know which particular concept to select or what kind of concept I to construe as the referent of a particular predicate-expression in a particular context? Without at least a sketch of an answer to this question all talk of functions is no more than a triviality.

In the simplest case, both the relevant context-model and the concept are already present in discourse. This case we find I typically exemplified in question-answer pairs. If I ask you "Have you fed the cat this morning?", and you reply "Yes, I have fed the cat this morning", or "Yes, I have", or simply "Yes", and there is no doubt about your utterance being intended as an answer to my question, then your utterance links up directly to the context-model as it stands after you have interpreted my question. The concept of having fed the cat this morning that is needed in the interpretation of your utterance is already present in this context-model: it must be identical to the concept that resulted from the interpretation of my question. We can talk, in these cases, about *anaphoric* interpretation of the predicate (cf. Bosch, 1980, 1983 § 3.2). An anaphoric interpretation of a predicate-expression is always required where the expression, without any change in interpretation, could be replaced by a verbphrase anaphor (such I as "have" in the above example, or a "so-do"-construction, etc.). Verbphrase anaphors may be used, as is shown in Bosch (1979), if and only if the concept referred to by the anaphor is identical to the concept referred to by the antecedent verbphrase. If, to take the above example, it is clear that the question was about whether the cat was given its usual milk-and-meat breakfast, the concept that is the referent of the corresponding expression contains this information. Accordingly, you could not reply "Yes, I have" (or in any other of the above forms) if, in fact, you just gave the cat some milk, some bisquits, or anything else that is not the cat's usual milk-and-meat breakfast (cf. also Ziff, 1973).

In this simple type of case then, there is neither a problem about determining the right sort of context-model for the interpretation nor about finding the concept referred to by the relevant predicate-expression with respect to the context-model at hand. Both are already given with the interpretation of the previous utterance. A more detailed account of anaphoric interpretation that actually delivers the relevant function in a calculable form would require a more or less fully worked out apparatus of syntactic and semantic (including pragmatic) description and is part of a theory of anaphora. It should be plain that we cannot here embark on even the sketch of such an account (but cf. Bosch, 1980, 1983).

Plainly, anaphoric interpretation is not the only form of interpretation we are concerned with. Even if there is a preceding context-model (and there always is one for an utterance in a coherent text or discourse) the concept we need, in order to interpret a particular predicate in

⁷ The fundamental notion exploited in this section, that understanding an utterance is to change one's representation of the context (i.e. one's context-model) was first put forward, as far as my own work is concerned, in Bosch (1973), although in a more primitive form than we are assuming here. The only earlier reference I am aware of is Ballmer (1972), which was developed independently but shares essential features with my own proposal. Cf. for later developments of Ballmer's proposal Ballmer (1978, 1979). The most explicit and up to date presentation of my own ideas on this point is now to be found in Bosch (in press). It should of course be acknowledged that meanwhile similar notions are very wide spread, often due to independent developments.

the utterance, may not yet be part of the context-model. All we have at our disposal in such a case is the previous context-model, the actual predicate-expression, possibly some knowledge of the actual context in rebus of which the context-model is a representation, and our ordinary background knowledge, including some knowledge of the language. The knowledge of the language may contribute, in particular, a history of earlier uses of the same predicate-expression with (an abstraction or generalization of) the concepts that have been used on earlier occasions as referents for that expression. Although this latter kind of information is certainly indispensable for the understanding of natural language utterances in the ordinary way, it can never offer any certainty of the kind anaphoric interpretation can offer. The abstractions or generalizations from concepts that have once been linked with a particular predicate are what has been called *stereotypes*. And all they can give us are default values for the interpretation of predicates: the concepts they give us are composed of a number of parts, each and all of which may be overwritten as soon as more direct information about the interpretation of the actual predicate-occurrence becomes available.

Apart from stereotypes though, there is another device that offers more certainty, although it offers less information. This device is a structural factor in the change of context-models. We assume that the interpretation of an utterance links up to a context-model present at the relevant moment and that it yields another context-model, which is a slightly modified version of the preceding one. That is, utterances are functions from context-models to context-models. In order to allow an utterance, however, to change our context-model, we must, preliminarily, assume that it is true (we limit our considerations here to assertive utterances). Now suppose, we are dealing with a simple utterance of a nounphrase-verbphrase structure and the referent of the nounphrase is already given in the preceding context-model, whereas there is no anaphoric interpretation for the predicate. What then must we minimally assume if we assume this utterance to be true? And hence: what information about the referent of the nounphrase (which is already in the context-model) must be added to the context-model in order to yield the new context-model? If we call the referent of the noun-phrase “a”, and the verbphrase is “F”, then we know at least that a has a property that is referred to in the object language by “F”. And of this property we know, again, its expression in the object language (i.e. “F”) and that *a* is in its extension. And this much (or rather: this little) we know for certain.

In the above account, we have tried to expose no more complexity than is absolutely essential for our task in connection with the paradox of the heap. Let us, finally, illustrate our account by an equally simple (and still simplified) example. Take the utterance-types “Fred is asleep” and “He is asleep” as they might occur in answer to a question like “Where is Fred?”. Here “Fred” and “He” would be interpreted anaphorically, i.e. with reference to the individual already present in the context-model that results from the interpretation of the question. Let us assume that there is no immediately preceding discourse that gives an anaphoric interpretation to “is asleep”. What then do we put into the new context-model over and above what it already contains after the interpretation of the question? We add a new property, call it *G*, that is specified as a pair consisting of the name of the property in the object language and the extension of the property, i.e. <“is asleep”, {*f*}>, for “*f*” being a constant representing the individual referred to by the name “Fred” in the object-language. Furthermore, we can add to the representation of Fred in the context-model the fact that *G* is a property of Fred’s. Clearly, this model, just because it is a minimal model, is a model for a large class of contexts. But it contains all we can reasonably claim to know for certain after the interpretation of the above utterance. Perhaps, if there are likely or obvious identifications of poorly specified individuals or poorly specified properties or concepts in a context-model with more fully specified individuals or concepts in the background knowledge of the listener, he may draw in the fuller

specifications from there. But these are matters we need not be concerned with in the present context.

5. The Sorites Paradox Resolved

In this section we want to demonstrate for the case of the *Sorites* paradox that seeming vagueness can be accounted for in terms of context-dependence.

For the advantage of greater clarity we shall consider the *Sorites* in the following form and not in the classical formulation of Eubulides:

- premisses:*
- (i) 1 is a small number.
 - (ii) If n is a small number, so is $n+1$.

conclusion: Any natural number $n \geq 1$ is small.

Let us note, to begin with, that the predicate “is a small number” selects rather different concepts in different contexts. If at the staff meeting, out of twenty members of the department, five are absent, one may perhaps find this too many to say that “a small number” of people are absent; in particular, if it is a meeting that has been scheduled in order to decide upon matters like the implementation of a thirty percent cut in the department’s budget. Still, five may be a small number if it is one of the regular meetings with nothing important on the agenda. If a government politician admits that also this month there was “still a small number of unemployed”, he may be talking about several hundred thousand or even several million people, depending on the size of the workforce and the state of the economy. If “a small number of students” in a twenty thousand students university are dissatisfied with their courses, this may be up to a couple of thousand students perhaps. If a small number of students in the same university are said to be ex-convicts, and what is meant is a couple of thousand, one may find the expression “small number” a gross understatement.

In this respect the first premiss is highly misleading: it suggests that the number 1 is a small number *tout court*, quite independent of the context. The predicate “is a small number” here C seems to have lost its ostensive character. It is implicitly assumed to be defined for all contexts, and for all contexts in the same way; just as if there was nor more than one concept of a small number. Still, premiss (i) need not lead into trouble. It could just happen that, despite differences in the various concepts of a small number, all concepts apply to the number 1.

In premiss (ii), however, the neglect of context-dependence of the interpretation of the predicate has more serious consequences. We may assume that the two occurrences of the predicate both refer to the same concept whenever the premiss is applied to a particular number in a particular context. The use of the verbphrase anaphor “so is” in the English formulation, which depends upon conceptual identity, would support this assumption (cf. Bosch, 1979). But we may not assume that this one concept of a small number will be the same one on each application of the premiss to different numbers in different contexts. And only if we make this latter assumption, we can re-apply premiss (ii) to its own results, i.e. apply it recursively. And only then we get the unacceptable conclusion.

To make our point somewhat more explicit, let us take a look at a formulation of the premisses in predicate-calculus.

- (i) IS-A-SMALL-NUMBER(1)
- (ii) $\forall n (\text{IS-A-SMALL-NUMBER}(n) \rightarrow \text{IS-A-SMALL-NUMBER}(n+1))$

From these formulations we can indeed infer the unwanted conclusion that any natural number $n \geq 1$ is small, that is

$$(iii) \quad \forall n \text{ (IS-A-SMALL-NUMBER}(n))$$

At the roots of this inference lies of course the use of predicate *constants* in the above formulations. Using predicate constants in the formalization means to ignore the ostensive character of the English predicate-expression “is a small number”, that is the context-dependence of its interpretation.

In order to account for the context-dependence of the predicate’s interpretation, we must take the predicate-expression rather as a function from context-models to concepts, and not as a constant. And, in accordance with our considerations in Section 4, we must: take the premisses not as abstract propositions but as utterances j that change context models. 1

If we take $\langle k \rangle$ as the semantic type of context-models and $\langle i \rangle$ as the semantic type of individuals, the following formulation gives a more adequate rendering of the premisses that is in agreement with our above considerations:

$$(i) \quad \exists c_{\langle k \rangle} (1_{\langle i \rangle} \text{ IS-A-SMALL-NUMBER}_{\langle k, \langle i, k \rangle \rangle} c_{\langle k \rangle})$$

$$(ii) \quad \forall c_{\langle k \rangle} \forall n_{\langle i \rangle} \quad ((n_{\langle i \rangle} \text{ IS-A-SMALL-NUMBER}_{\langle k, \langle i, k \rangle \rangle} c_{\langle k \rangle}) \rightarrow \\ ((n+1)_{\langle i \rangle} \text{ IS-A-SMALL-NUMBER}_{\langle k, \langle i, k \rangle \rangle} c_{\langle k \rangle}))$$

What premiss (i) says is that there is at least one context-model in respect of which the number 1 has a particular property: the property that is the value of the predicate “is a small number” for that context-model.

Contexts that satisfy premiss (i) are partially modelled by the context-model CM_i :

$$CM_i = \langle I, C, P \rangle$$

$$I = \{1\}$$

$$C = \{1^*\}$$

$$P = \{P_1\}$$

$$1^* = \langle \text{“1”}, \{P_1\} \rangle$$

$$P_1 = \langle \text{“is a small number”}, \{1\} \rangle$$

That is, CM_i is a triple of a set of individuals, I , a set of characters, C , and a set of properties, P , such that I contains the individual 1, C contains the character of 1, i.e. 1^* , and P the property P_1 . The property P_1 in turn is given as the pair consisting of the object-language name of P_1 and the set that forms the extension of P_1 . The information on the individual 1 is represented in the individual’s *character*, 1^* , which is a pair of the object-language name of the individual and the set of the individual’s properties. In general, if “ a ” is an individual constant, “ a^* ” is the constant for the character of a .

Now, premiss (ii) says that for any context-model c and any natural number n , if n has the property that is the interpretation of the predicate-expression “is a small number” in a particular context-model c , then also the number $n+1$ has the same property.

Suppose, we now choose a context in which premiss (i) is satisfied. Any such context is partially modelled by CM_i above. In a next step, we apply premiss (ii) to CM_i . If the premiss is true, then any such model may be extended to a model CM_{i+1} that contains, additionally, the individual 2, which is specified as to its name and as to its property of also being a small number (in the same sense as 1 is a small number), and it contains an extended definition of

the property P_1 (i.e. of being a small number in the relevant sense) so as to cover the number 2 as well as 1. CM_{i+1} would then be given as follows:

$$\begin{aligned}
 CM_{i+1} &= \langle I, C, P \rangle \\
 I &= \{1,2\} \\
 C &= \{1^*,2^*\} \\
 P &= \{P_1\} \\
 1^* &= \langle \text{"1"}, \{P_1\} \rangle \\
 2^* &= \langle \text{"2"}, \{P_1\} \rangle \\
 P_1 &= \langle \text{"is a small number"}, \{1,2\} \rangle
 \end{aligned}$$

The inference from “1 is a small number” to “2 is a small number” thus is valid also according to our reconstruction. Analogously, of course, any inference of the form “if n is a small number, so is $n+1$ ” would come out true under our reconstruction. And that is fine. But what happens to the possibility of recursive application of premiss (ii), and hence to the threat of paradox?

We have started from premiss (i) and we have applied premiss (ii) to the context-model resulting from premiss (i), thus arriving at CM_{i+1} and the conclusion that 2 is a small number. Let us continue and try to apply premiss (ii) for a second time, thus showing that 3 is a small number whenever 2 is.

We must, start, however, again, by constructing a partial model for all contexts with respect to which 2 is a small number. Let us call this model CM_j :

$$\begin{aligned}
 CM_j &= \langle I, C, P \rangle \\
 I &= \{2\} \\
 C &= \{2^*\} \\
 P &= \{P_2\} \\
 2^* &= \langle \text{"2"}, \{P_2\} \rangle \\
 P_2 &= \langle \text{"is a small number"}, \{2\} \rangle
 \end{aligned}$$

By application of premiss (ii) we can extend CM_j to CM_{j+1} :

$$\begin{aligned}
 CM_{j+1} &= \langle I, C, P \rangle \\
 I &= \{2,3\} \\
 C &= \{2^*,3^*\} \\
 P &= \{P_2\} \\
 2^* &= \langle \text{"2"}, \{P_2\} \rangle \\
 3^* &= \langle \text{"3"}, \{P_2\} \rangle \\
 P_2 &= \langle \text{"is a small number"}, \{2,3\} \rangle
 \end{aligned}$$

We see, as expected, that we can go from “2 is a small number” to “3 is a small number”, just as we could go from “1 is a small number” to “2 is a small number”. But we cannot without further ado, go from CM_{i+1} to CM_j and hence, we cannot guarantee that the property referred to by the predicate-expression “is a small number” is the same in both applications of the

second premiss. In fact, as specified in the partial models CM_{i+1} and CM_j , the two properties are different: one has the extension $\{1,2\}$ and the other has merely the extension $\{2\}$. This need not mean that the two properties are different, for we are concerned with partial models. There may be a property of which P_1 and P_2 are partial specifications. But it follows from nothing in our reconstruction that there must be such a property, just as it does not follow that there must be a context that is partially modelled both by CM_{i+1} and by CM_j .

It might be thought that we should not construct a new CM_j when we apply the second premiss to the number 2, but that we should immediately link up to CM_{i+1} . But this would plainly be unwarranted. There is no indication whatever that we are concerned with the same context on both applications of the second premiss, and hence there is no reason to assume that CM_{i+1} and CM_j should be models of the same context. And whenever we do not have a given context to which the statement “ n is a small number” is to be applied, we have to start out by constructing a minimal model for the interpretation of the first clause of the second premiss, i.e. a model that contains no more information than is actually given in the clause. Hence, we may not link up with our second application of premiss (ii) to CM_{i+1} but we have to construct a new minimal model: CM_j .

Now, there are of course contexts for which the number 1, 2, and 3 (and perhaps more) are small numbers. But this says nothing against the results of our reconstruction. In such cases, we would know from our knowledge of the context under consideration that e.g. CM_i , CM_{i+1} , CM_j , and CM_{j+1} are *in fact* all partial specifications of one and the same context and hence that they may be merged into one fuller context model, as long as we are concerned with that particular context. But note that no such information is contained in the two premisses and that it does, of course, not hold in general.

The crucial features of our reconstruction of the Sorites-premisses then is, to sum up, that the two occurrences of the predicate “is a small number” (or “is a heap” or what not) in the second premiss must have the same property as their value (that is why the premiss is true); but on any new application of the second premiss to a particular number (or to any object that is a heap, for that matter) the property of being a small number (or a heap) that is at issue need not be identical to any other property of the same name that occurs in any other application of the premiss (that is why recursive application of the second premiss is excluded, and hence the paradox does not arise)⁸.

Intuitively, the solution presents itself as follows. Whenever, in a particular context, we are presented with a number and have to judge whether, relative to the context at hand, it is a small number, we employ one fixed concept of a small number, i.e. the concept of a small number we find appropriate for that context. Now, if we decide for a number m that it is to count as small, the second premiss of the Sorites forces us to also count $m+1$ as small. But this does not bind us with respect to our decision about whether or not $m+1$ is a small number,

⁸ There are people, apparently, who think that premiss (ii) is false. They can demonstrate their claim by pointing out contexts in which there is an n that is a small number in the sense relevant for that context and an $n+1$ for which the same property does not apply. Of course, if we follow this line of arguing, the whole paradox does not arise. But also the by and large valid inferences of the form “if n is a small number, so is $n+1$ ” cannot be accounted for. We thought it more interesting to accept premiss (ii) and show that the paradox still need not arise. It will be clear however to the reader (after a bit of thought) that our approach of making the interpretation of predicates depend on context models is in principle capable of coping also with odd contexts of the kind suggested. We are not going into this matter however, because here our concern is with the reading of premisses (i) and (ii) on which they are both true and hence seem to lead to a paradox. Also premiss (i), of course, will not be true in “some contexts, like, as Alice ter Meulen suggested to me, the context of probability theory: as a probability measure, the number 1 is certainly anything but small.

when we are faced with that question independently, i.e. not in the context arising *after* the decision on the number m , but in the same context in which the first decision (on m) was made. For only if we start out from that context, we may assume that we are concerned with the same context for both the decision on m and the decision on $m+1$. And only if the context is the same, we can be sure that we are concerned with the same concept.

Accordingly, in a particular context we may make, independently of each other, the following decisions:

- (i) i is a small number, hence also $i+1$ is small.
- (ii) $i+1$ is a small number, hence also $i+2$ is small.
- (iii) $i+2$ is not a small number (and hence nothing follows for $i+3$).

The point to note here is that there is no contradiction between (ii) and (iii), because, although the *initial* context, in which the decisions about i , $i+1$, and $i+2$ are made, is identical in (i)-(iii), the contexts *resulting* from (i) and (ii) are different from the initial context: the definition of the property of being a small number in each of these cases gets changed in the course of the interpretation of (i) and (ii). Hence, in (ii) the concept under which $i+2$ is said to fall, is different from the concept under which, in (iii), $i+2$ is said not to fall.

There is one last issue we have to address: the question of borderline-vagueness. One might think of situations like those we mentioned at the very beginning of this paper of, say, a departmental meeting in a department of twenty members, and one might have in mind one particular meeting and then ask oneself: which would be the greatest number of absentees one could still call a small number? Or one might ask oneself about a particular number, say 5, whether this would still be a small number in that context. If I was asked this question, I think I would hedge, even knowing everything about the context one could possibly know. But does this not mean that, although the above account may resolve the paradox of the heap, it does not succeed in reducing vagueness to context-dependence?

No, it does not mean that. If the context in respect of which a particular concept is to be determined is completely known (whatever that may mean), and no completely defined concept is forthcoming, then this means quite simply that already the very question, e.g. whether or not 5 is a small number with respect to that context, does not make sense. And if someone asks such a question in all seriousness, then we are probably mistaken about the context-model he assumes and we have to ask him what exactly he means, or what the point of his question is, so that we will get a better idea of his context-model. But once there is no doubt about the correctness and completeness of our knowledge of the speaker's context-model, and a completely defined concept is still not forthcoming, we are back where we already were and have to count the question as nonsensical; similarly nonsensical as the question "Have you stopped beating your wife?" addressed to someone of whom it is known that he has never indulged in such eccentricities.

The function of hedges, however, like "5 is *not really* a small number *any more*" or "5 is *hardly* a small number of absentees" and the like, is not to cover up nonsense. Hedges are appropriate when we cannot be bothered to get a full picture of the relevant context-model. They save us the immense trouble of attempting a full understanding and thus have a crucial place in natural language communication. They are means for deliberately introducing vagueness as a means of quick and efficient understanding on the appropriate or desired level of (im-)preciseness. And in describing this function of vagueness, specific vagueness-logics have their place. Not though in any attempts of achieving conceptual clarity about the role of vagueness in natural language semantics.

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