

Vagueness and Context

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What I am going to argue

The Sorites Paradox (e.g. for gradable adjectives) arises only on the naïve assumption that
lexical expressions have a constant denotation, identical for all utterance contexts.
(words map onto concepts 1:1)

This assumption is hard to accept in view of what most semanticists currently believe, and is untenable in view of many results from empirical work on utterance comprehension.

The agenda

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1. The sketch of a formal reconstruction of the Sorites
2. Vagueness is context dependence
3. Context dependence of semantically incomplete expressions

Vagueness

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Philosophers tend to ascribe vagueness to "predicates" and illustrate the behaviour of "vague predicates" with the help of natural language expressions.

Somehow, though, it is assumed that these "predicates" are not the same as the linguistic expressions, but rather their "meanings", or "concepts" of some kind.

The Sorites reconstructed (1 & 2)

(1) *Common English formulation of the Sorites:*

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- (i) *1 is a small number.*
 - (ii) *If n is a small number, so is $n+1$.*
-
- (iii) *Any natural number is small.*

(2) *A formulation in QPC (using the "predicate" SMALL-NUM):*

- (i) SMALL-NUM(1)
 - (ii) $\forall n$ (SMALL-NUM(n) \vdash SMALL-NUM($n+1$))
-
- (iii) $\forall n$ (NATURAL-NUMBER (n) \vdash SMALL-NUMBER(n))

lemmas required:

- (a) NATURAL-NUMBER(1)
- (b) $\forall n$ (NATURAL-NUMBER(n) \vdash NATURAL-NUMBER($n+1$))

The Sorites reconstructed (3)

(3) *Taking "predicates" as denotations of linguistic expressions:*

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- (i) $1 \in \llbracket \textit{is a small number} \rrbracket$
 - (ii) $\forall n$ ($n \in \llbracket \textit{is a small number} \rrbracket \vdash (n+1) \in \llbracket \textit{is a small number} \rrbracket$)
-
- (iii) $\forall n$ ($n \in \llbracket \textit{is a natural number} \rrbracket \vdash (n \in \llbracket \textit{is a small number} \rrbracket)$)

lemmas required:

- (a) $1 \in \llbracket \textit{is a natural number} \rrbracket$
- (b) $\forall n$ ($n \in \llbracket \textit{is a natural number} \rrbracket \vdash (n+1) \in \llbracket \textit{is a natural number} \rrbracket$)

Saving the Sorites from the philosophers

Viewing "predicates" as denotations of the NL predicate expressions is a step in the right direction:

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Instead of talking indiscriminately of "predicates" we can now distinguish linguistic expressions from their denotations and can model

the denotation of (an occurrence) of a linguistic expression in an utterance as a function of the utterance context.

Thus we can accommodate the observation that an expression *does not have the same denotation in all contexts*:

What is a "small number" in one context, may not be a small number in another.

variability in adjective denotation

What is a small number of students?

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when you are talking about the number students dissatisfied with their courses in a 40.000 student university?

- perhaps a few hundred?

when you are talking about the number students in the same university who are ex-convicts?

- perhaps a dozen?

Our answer still allows for quite a bit of tolerance:

- we have only given a vastly underspecified representation of the utterance context,
- and even in a fully known context we still want to keep some tolerance: That's the point of a gradable adjective.

The Sorites reconstructed (4)

(4) *Talking about linguistic expressions and making their denotations depend on utterance context (with the exception of ¹⁰ "is a natural number"), the Sorites argument looks like this:*

- (i) $\exists c (1 \in \llbracket \text{is a small number} \rrbracket^c)$
- (ii) $\forall c \forall n (n \in \llbracket \text{is a small number} \rrbracket^c \vdash (n+1) \in \llbracket \text{is a small number} \rrbracket^c)$

- (iii) $\forall n (n \in \llbracket \text{is a natural number} \rrbracket \vdash \forall c (n \in \llbracket \text{is a small number} \rrbracket^c))$

But how can we derive (iii) from (i) and (ii)?

lemmas required as before: (a) & (b)

- (a) $1 \in \llbracket \text{is a natural number} \rrbracket$
- (b) $\forall n (n \in \llbracket \text{is a natural number} \rrbracket \vdash (n+1) \in \llbracket \text{is a natural number} \rrbracket)$

The Sorites reconstructed (4)

- (i) $\exists c (1 \in \llbracket \text{is a small number} \rrbracket^c)$
- (ii) $\forall c \forall n (n \in \llbracket \text{is a small number} \rrbracket^c \vdash (n+1) \in \llbracket \text{is a small number} \rrbracket^c)$ ¹¹

lemmas (a) & (b)

- (a) $1 \in \llbracket \text{is a natural number} \rrbracket$
- (b) $\forall n (n \in \llbracket \text{is a natural number} \rrbracket \vdash (n+1) \in \llbracket \text{is a natural number} \rrbracket)$

by (a) and (b) we derive the intermediate conclusion (α):

$$(\alpha) \forall c \forall n (n \in \llbracket \text{is a small number} \rrbracket^c \vdash n \in \llbracket \text{is a natural number} \rrbracket)$$

in order to derive (iii):

- (iii) $\forall n (n \in \llbracket \text{is a natural number} \rrbracket \vdash \forall c (n \in \llbracket \text{is a small number} \rrbracket^c))$

we need, in addition, the following lemma:

- (c) $\forall n ((\exists c (n \in \llbracket \text{is a small number} \rrbracket^c)) \vdash \forall c (n \in \llbracket \text{is a small number} \rrbracket^c))$

i.e., we must keep the denotation of "is a small number" constant.

The Sorites reconstructed (4)

- (i) $\exists c (1 \in \llbracket \text{is a small number} \rrbracket^c)$
(ii) $\forall c \forall n (n \in \llbracket \text{is a small number} \rrbracket^c \vdash (n+1) \in \llbracket \text{is a small number} \rrbracket^c)$ ¹²
(iii) $\forall n (n \in \llbracket \text{is a natural number} \rrbracket \vdash \forall c (n \in \llbracket \text{is a small number} \rrbracket^c))$

lemmas (a) & (b)

(a) $1 \in \llbracket \text{is a natural number} \rrbracket$

(b) $\forall n (n \in \llbracket \text{is a natural number} \rrbracket \vdash (n+1) \in \llbracket \text{is a natural number} \rrbracket)$

intermediate conclusion (α):

(α) $\forall c \forall n (n \in \llbracket \text{is a small number} \rrbracket^c \vdash n \in \llbracket \text{is a natural number} \rrbracket)$

the critical lemma:

(c) $\forall n ((\exists c (n \in \llbracket \text{is a small number} \rrbracket^c)) \vdash \forall c (n \in \llbracket \text{is a small number} \rrbracket^c))$

By stating that if something is a small number in one context, it is a small number in all contexts, (c) begs the question:

We can derive the Sorites conclusion only on assumption that the expression "is a small number" has the same denotation in all utterance contexts.

Intermediate summary

1. The Sorites paradox does not arise, unless we postulate that the adjective small has the same denotation in each of the repeated applications of premise (ii).
2. The very source of the paradox is in intuitively mixing up the expression and its denotation, word and concepts.
3. If we use an adjective that is plausibly stable across contexts, like vertical (a gradable absolute adjective (Kennedy & McNally) the paradox does not even arise intuitively.

We may use vertical in a strict sense, with no tolerance, or with a defined tolerance, as in the building industry ($\text{tol} < 1/1000$), in neither case do we get a Sorites effect.

Intermediate summary

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(i) *A 5 meter high pillar deviating 1 mm from the vertical is vertical.*

(ii) *A 5 m high pillar deviating 1 mm more from the vertical than a vertical pillar is still vertical.*

(iii) Any 5m high pillar is vertical

3. If we use an adjective that is plausibly stable across contexts, like vertical (a gradable absolute adjective (Kennedy & McNally)) the paradox does not even arise intuitively.

We may use vertical in a strict sense, with no tolerance, or with a defined tolerance, as in the building industry ($\text{tol} < 1/1000$), in neither case do we get a Sorites effect.

No Sorites with context-dependent denotations

condition of premise (ii)

"if 1 is a small number"

$[[1]] \in [[\text{is a small number}]]^{c_i}$

$[[1]] = 1$

$\{1\} \subset [[\text{is a small number}]]^{c_i}$

consequent of premise (ii):

$[[2]] \in [[\text{is a small number}]]^{c_i}$

$[[2]] = 2$

$\{1,2\} \subset [[\text{is a small number}]]^{c_i}$

condition of premise (ii)

"if 2 is a small number"

$[[2]] \in [[\text{is a small number}]]^{c_k}$

$[[2]] = 2$

$\{2\} \subset [[\text{is a small number}]]^{c_k}$

consequent of premise (ii):

$[[3]] \in [[\text{is a small number}]]^{c_k}$

$[[3]] = 3$

$\{2,3\} \subset [[\text{is a small number}]]^{c_k}$

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Within each application of premise (ii) c is constant and so is the denotation of "small"

But between two applications c_i may or may not be the same as c_k !

Intermediate summary

1. The Sorites paradox does not arise, unless we postulate that the adjective *small* has the same denotation in each of the repeated applications of premise (ii).
2. The very source of the paradox is in intuitively mixing up the expression and its denotation, word and concepts.

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Context dependence of linguistic comprehension: The undisputed case

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So-called *indexicals*, i.e., referential expressions whose reference depends on information in the utterance situation:

I, you, here, now, yesterday, tomorrow, this, that,...

Truth-conditions for a sentence containing one or more indexicals depend on the utterance context.

David Kaplan's architecture for indexicality

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compositionally
constructed
sentence
meanings

Characters: Contexts → Contents

propositions,
truth
conditions,
what is said

Utterance
meanings

what is meant

Context-dependence of other than indexical expressions...

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But what about other than referential expressions
(semantically unsaturated expressions)?

Sometimes it would seem that also predicates behave like
indexicals:

Point of view

My apartment is nearby

The department is on the left side of the street

Comparison class / standard

He is a tall basketball player

He is a tall jockey

Processing of the German definite determiner

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default nouns:

die Giraffe

the giraffe

die Rakete

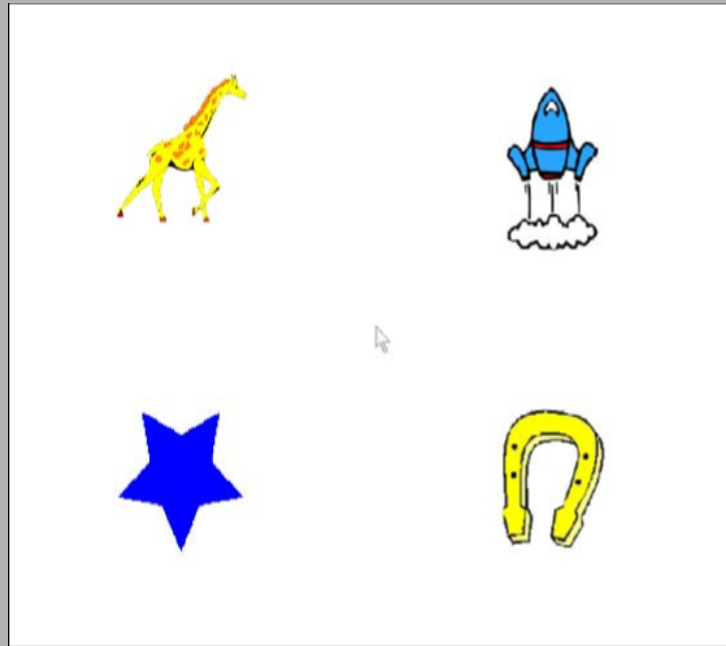
the rocket

der Stern

the star

das Hufeisen

the horse shoe



Klicken Sie auf die blaue Rakete.

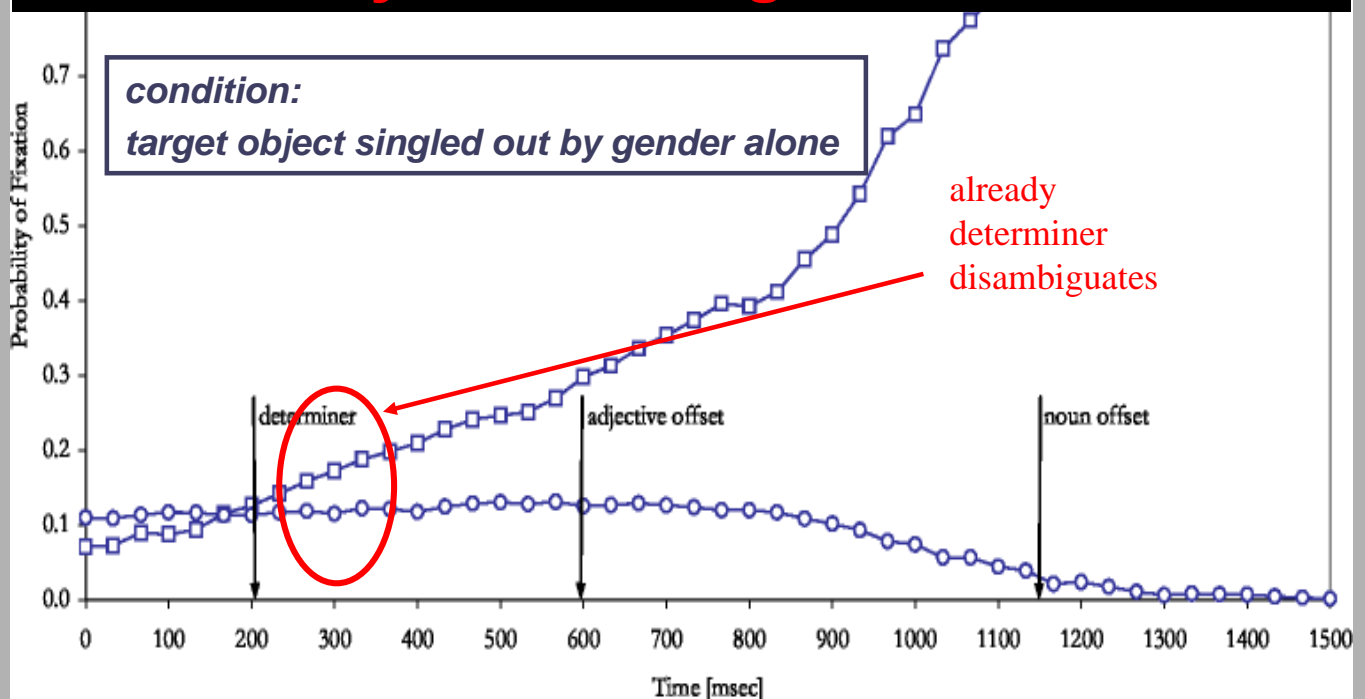
click on ... [followed by a def. determiner, adjective, and noun]

Hartmann 2005

Processing of the German definite determiner

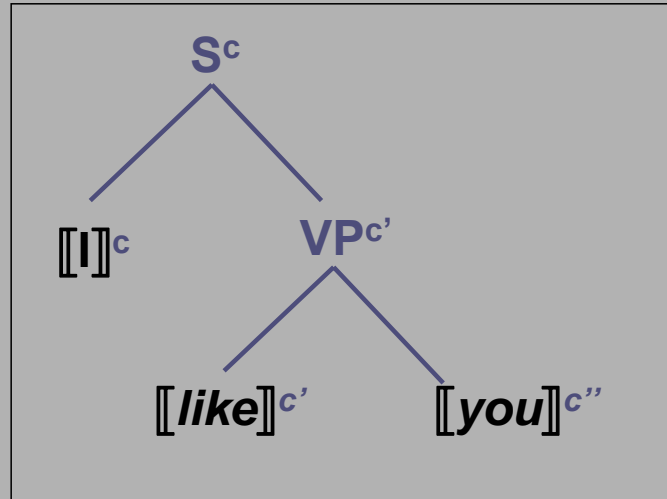
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subjects decide about reference as soon as they have enough information



Immediate context influence

This invites an application of Kaplan's conception to meanings of constituents - ultimately lexical items - as they become available consecutively during sentence processing. 22



Productive modification

The kind of contextual influence we are interested in

- is a form of modulation that is made and understood automatically and with no effort,
- remains unnoticed by the language user, and
- can yield, in principle, infinitely many variants of arbitrarily fine granularity

Typical cases:

{My watch / The tap / The lecture...} is running.

{Fred / my computer / my solution} is working

There is a petrol station nearby.

The small car is a Mercedes.

Productive modification (i)

where the semantic values of argument expressions seem to contain all information relevant

(1) *to cut*

hair, bread, cake, lawn, ...

(2) *to open*

book, letter, door, bottle, buffet, ...

(3) *to run*

tap, water, clock, show, dog, ...

(4) *a small*

elephant, dog, horse, ...

- What if you only have the argument *expressions* & not their *denotations*?
- *How many different types* of arguments are there? What if the list is not finite?

Productive modification (ii)

where the semantic values of an implicit argument is sensitive to context

(1) *nearby*

location

(2) *enemy*

person

(3) *left*

point of view

- How do you *find the relevant information in the context*?
- How do we know *what expressions* have implicit arguments?

Productive modification (iii)

where neither explicit nor implicit arguments help:
contextual variation in the denotation of "work"

Where is Fred?

(1) *He's working.*

$WORK_i(\text{fred}) \rightarrow \varphi (\text{LOCATION}(\text{fred}))$

How can Fred afford these expensive holidays?

(2) *He's working.*

$WORK_j(\text{fred}) \rightarrow \psi (\text{WEALTH}(\text{fred}))$

Can I speak to Fred, please?

(3) *He's working.*

$WORK_k(\text{fred}) \rightarrow \sigma (\text{AVAILABILITY}(\text{fred}))$

Productive modification (iii)

The variation in the denotation of *work* is inferentially and hence truth-conditionally relevant.

Nothing follows about Fred's *location* when

Fred is working

is an answer to

How can Fred afford these expensive holidays?

The variation is stable within the utterance context.

Fred is working and so is Pete.

cannot be interpreted as, e.g.,

Fred is in his office and Pete can afford expensive holidays.

What is this variation a variation of ?

- not of lexical meanings (characters)
(because the variation is productive and correlates with variation in the context)

character: context → content

- but of semantic values (contents, denotations)

Contextual denotations of "incomplete" expressions are

Contextual Concepts (CCs)

(Cf. Frege's idea that the values of "predicates" are concepts.)

What are Contextual Concepts ?

Contextual Concepts are

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- **partial truth functions that are defined for all arguments in the intended context**
- **constructed on the fly in the course of language production and comprehension**
- **linguistically "real":**
They define the required notion of identity in VP anaphora, coordination, question-answer coherence; they define units of counting.
- **psychologically "real" in comprehension processes**