Deontic Radical Inquisitive Semantics (DRIS)

January 24, 2012

Introduction

(1) A country may establish a research center or a laboratory.

When (1) is international law, it gives permission to establish a research center and it gives permission to establish a laboratory. Note that it differs from conjunction because it does not necessarily give permission to establish both. Free choice disjunction has become one of the better documented puzzles in semantics since it was investigated in [Kamp, 1973].

The approach to free choice outlined here is based on an investigation of World Trade Organisation dispute texts, which documents discourse in a deontic setting between a complainant, respondent and a panel of judges. The results of [Aher, 2010] suggest that judges organize legal discourse around central questions on whether specific laws have been violated. The complainant has the burden of proof to demonstrate that an act was performed and to specify the law that it violated. The respondent has to either deny the act or break the link between the act and the law.

A violation-based deontic logic gravitates around the question whether an act violates a specific law. A permission sentence in a law text provides information on what is not a violation. Legal discourse that gravitates around violations suggests an approach based on Anderson's reduction of a permission utterance $\Diamond p$ to $p \rightarrow \overline{v}$ [Anderson, 1967] which inspired the current violation-based approach to the free choice permission puzzle implemented in the framework of inquisitive semantics.

Previous accounts

[Zimmermann, 2000] provides a solution through positing a pragmatic mechanism that reinterprets disjunction as a conjunctive list of epistemic possibilities: $\Diamond A \wedge \Diamond B$. Unfortunately, such an approach does not explain why this reinterpretation arises in those contexts nor does it account for the intuition that free choice disjunction does not necessarily give permission to do both A and B. Furthermore, [Alonso-Ovalle, 2004, Simons, 2005b, p. 8] draws attention to the fact that

disjunctive permission in a downward entailing context once again behaves standard - an effect Zimmermann fails to predict.

Many following accounts accepted that free choice is essentially a pragmatic effect and suggested that the phenomenon is an implicature. These accounts include [Schultz, 2005, Eckardt, 2007, Fox, 2007] and the game theoretic implicature account by [Franke, 2009]. As approaches to free choice have been extensively discussed in the literature, for example by [Schultz, 2003] or more recently in [Barker, 2010], we will concentrate on examining [Eckardt, 2007] to expatiate on general issues with implicature-based solutions.

Eckardt derives the free choice effect utilizing an implicature through the maxims of manner and quality.

1. Informed speaker uses disjunction: $\Diamond(\varphi \vee \psi)$.
2. Either disjunct would be more economical.
3. Infer that permissions are best described by disjunction because either disjunct would be false.
4. Free choice effect: There must be some worlds where $\Diamond\varphi \wedge \overline{\Diamond\psi}$ and others where $\overline{\Diamond\varphi} \wedge \Diamond\psi$.

The weakness of this account lies in step 4 as the intuition behind the deontic free choice effect is that the speaker's information state supports only worlds in which A and B are permitted. The reason why $\Diamond\varphi \wedge \overline{\Diamond\psi}$ and $\overline{\Diamond\varphi} \wedge \Diamond\psi$ worlds are the case is that there exist some worlds in the speaker's information state in which either φ or ψ is not permitted. But this is contrary to the intuition outlined above.

[Simons, 2005b, p. 14] argues generally against implicature based accounts on the grounds that there does not seem to be a distinction between what is said and what is implicated in examples such as (1). Compare this to a classic example of generalized implicature from [Grice, 1989b, p. 32].

(2) X is meeting a woman this evening.

Grice states that such a statement generally implicates that the woman being met is not X's wife, mother, sister, etc. Thus, there exists a clear distinction between that which is said (X will meet a woman) and that which is implicated (X will meet a potential romantic acquintance). The lack of such distinctions in free choice sentences poses a challenge to any implicature based account.

[Barker, 2010, p. 16] demonstrates that another marker of implicatures is visibly lacking, namely cancellability. Observe the following example.

(3) You may eat an apple or a pear, although in fact you may not eat an apple.

When an implicature is cancelled, the utterance only has the meaning of what is said. If (2) were cancelled by "... but it's only her mother." then the utterance would lose the implicature that the woman is a romantic acquintance. Yet, instead of reverting the phrase to that which is said as opposed to that which is implied, the added phrase in (3) appears to make the statement contradictory.

There appears to be another possible route for cancellation, which is to utter either of the following continuations.

(4) You may eat an apple or a pear, although in fact you may not eat both.

(5) You may eat an apple or a pear, although in fact I do not know which.

The consequence of uttering (4) does not cancel the free choice effect. Permission is given to eat an apple and permission is given to eat a pear. Yet, the continuation provides the additional information that eating both an apple and a pear is prohibited. This additional information does not conflict with free choice readings.

(5) intuitively suggests that the speaker is ignorant. The speaker has limited knowledge of the governing permissions and prohibitions, and utters the most helpful utterance available. It is well known in the literature that such utterances have standard disjuntive entailment relations, and it can easily be accounted for by inquisitive semantics.

[Barker, 2010] proposes a semantic approach similar to the one pursued here, by following [Kanger, 1971] in positing a normative ideality δ such that if φ is obligatory, then if φ then δ. This view is a contrapositive view of Anderson's reduction and, thus, is in no conflict with the current solution to the free choice puzzle at a foundational level. But in terms of details, the analysis of WTO examples in [Aher, 2010] suggests that legal reasoning does not concern idealities but rather violations. While this might be contingent on the deontic context, in terms of legal language, the violation-based solution remains preferable.

Furthermore, [Barker, 2010, p. 11] correctly notes that if obligation is rendered as "if φ then δ" then doing φ guarantees the ideal universe. In a standard model, this means that if you may eat an apple, then eating an apple and killing a postman will invariably lead to the ideal universe. Barker introduces a resource sensitive calculus to render such inferences invalid.

As will be demonstrated, Barker's approach seems intuitively to be on the right track, but it lacks certain aspects which will be included in the approach to follow. For example, a violation based system allows for inferences with different violations, such as when two different laws are relevant to judge a case. An ideality based model would require significant work to account for these cases.

Also, while Barker's account of the free choice effect is entailment based, he introduces pragmatics to attain the default reading of negated disjunctive permission sentences. It would be more aligned with his project to provide a fully semantic account. This observation, albeit not a counterargument in itself, also holds for the semantic account in [Aloni, 2007].

Deontic Radical Inquisitive Semantics

Negation of disjunctive permission utterances is one of the fundamental problems for many accounts on free choice and this fact is often taken as support for the idea that the phenomenon should be resolved by an implicature. We shall thus base the inquisitive deontic model on an independently motivated prior version of inquisitive semantics that focuses on the effects of negation - Radical Inquisitive Semantics. An earlier version of the language used here was developed and explored in [Sano, 2010]. Our proposal adds clauses for deontic permission and discusses entailment in the radical environment. Unlike [Groenendijk and Roelofsen, 2010], this version of Radical Inquisitive Semantics is restricted. Also note the definition for the negation of implication, the original motivation for such a formulation is discussed in [Groenendijk and Roelofsen, 2009b, pp. 18-23, 28-30].

We shall only consider a propositional language of a finite set of propositional variables and the operators: $\overline{\varphi}, \wedge, \vee, \rightarrow, \neg$. Negation is defined as $\overline{\varphi}$ and $\neg$ is added as classical negation for comparison purposes and will only play a limited role in the deontic story. We also need to define worlds as binary valuations for atomic sentences and states as non-empty sets of worlds. σ and τ are variables that range over states, w is the variable that ranges over worlds and W is the set of all (classical) valuation functions. Propositions expressed by sentences are defined through support. When a state supports φ then we write $\sigma \models^{+} \varphi$ and when a state rejects φ then we write $\sigma \models^{-} \varphi$.

Generally, v shall designate a specific law or regulation that is being violated. To account for different types of violations that can occur within a single legal framework, one can designate v_1, v_2, etc. for each specific violation. For example, v_1 may be taken as the proposition "Violation of law number 1 has occurred." As violation propositions are specific, violations can be reasoned about in the same manner as any other proposition. So the violation of one law does not lead to violations of other laws, nor does not violating one law save one from indictments due to other deeds.[1]

Definition 1. Radical inquisitive semantics.

1. $\sigma \models^{+} p$ iff $\forall w \in \sigma : w(p) = 1$
   $\sigma \models^{-} p$ iff $\forall w \in \sigma : w(p) = 0$

2. $\sigma \models^{+} \overline{\varphi}$ iff $\sigma \models^{-} \varphi$
   $\sigma \models^{-} \overline{\varphi}$ iff $\sigma \models^{+} \varphi$

3. $\sigma \models^{+} \neg\varphi$ iff $\forall \tau \subseteq \sigma . \tau \not\models^{+} \varphi$
   $\sigma \models^{-} \neg\varphi$ iff $\sigma \models^{+} \varphi$

[1] This formulation might yield interesting results with classic deontic logic puzzles, such as the Chisholm's paradox and the gentle murder paradox. Yet, as these fall out of the scope of describing the natural language semantics of permissive disjunction sentences, it will not be discussed in this article.

4. $\sigma \models^+ \varphi \vee \psi$ iff $\sigma \models^+ \varphi$ or $\sigma \models^+ \psi$
   $\sigma \models^- \varphi \vee \psi$ iff $\sigma \models^- \varphi$ and $\sigma \models^- \psi$

5. $\sigma \models^+ \varphi \wedge \psi$ iff $\sigma \models^+ \varphi$ and $\sigma \models^+ \psi$
   $\sigma \models^- \varphi \wedge \psi$ iff $\sigma \models^- \varphi$ or $\sigma \models^- \psi$

6. $\sigma \models^+ \varphi \rightarrow \psi$ iff $\forall \tau \subseteq \sigma.(\ \tau \models^+ \varphi$ implies $\tau \models^+ \psi)$
   $\sigma \models^- \varphi \rightarrow \psi$ iff $\exists \tau.(\tau \models^+ \varphi$ and $\forall \tau' \supseteq \tau.(\tau' \models^+ \varphi$ implies $\sigma \cap \tau' \models^- \psi))$

7. $\sigma \models^+ \Diamond\varphi$ iff $\forall \tau \subseteq \sigma.(\ \tau \models^+ \varphi$ implies $\tau \models^+ \overline{v})$
   $\sigma \models^- \Diamond\varphi$ iff $\forall \tau \subseteq \sigma.(\ \tau \models^+ \varphi$ implies $\tau \models^- \overline{v})$

Persistence, restriction and (counter-)possibilities

Definition 2. Propositions

1. $\|\varphi\|^+ := \{\tau \subseteq W \mid \tau \models^+ \varphi\}$
   $\|\varphi\|^- := \{\tau \subseteq W \mid \tau \models^- \varphi\}$

The model is persistent, as $\|\varphi\|^+$ and $\|\varphi\|^-$ are closed under $\subseteq$ ie. when a state supports $\|\varphi\|^+$ then so does each of its substates.

The clauses for possibilities and counter-possibilites differ from those in [Groenendijk and Roelofsen, 2009b] with respect to the addition of a filter that ensures that possibilities and counter-possibilities are maximal states that support or reject a sentence.

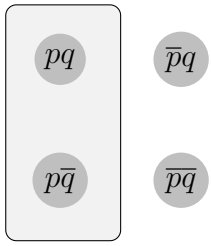
Definition 3. Maximality restriction
Given any $\chi \subseteq Pow(W)$, χ_{MAX} is defined as all the $\subseteq$-maximal elements of χ, ie. $\sigma \in \chi_{MAX}$ means that, for any $\tau \in \chi$ with $\sigma \subseteq \tau$, $\sigma = \tau$.

This allows us to define possibilities and counter-possibilities. We define for every sentence φ in our language, the proposition $\lceil\varphi\rceil$ expressed by φ, and the counter-proposition $\lfloor\varphi\rfloor$ for φ. Both $\lceil\varphi\rceil$ and $\lfloor\varphi\rfloor$ are sets of possibilities. We will refer to the elements of $\lceil\varphi\rceil$ as the possibilities for φ and to the elements of $\lfloor\varphi\rfloor$ as the counter-possibilites for φ.

Definition 4. Possibilities and counter-possibilities
$\lceil\varphi\rceil := \|\varphi\|^+_{MAX}$
$\lfloor\varphi\rfloor := \|\varphi\|^-_{MAX}$

Propositions and possibilities

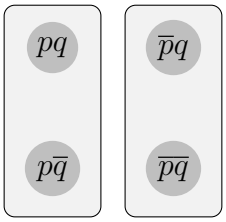
The picture that emerges builds on the classical notion that the meaning of a sentence is the proposition that it expresses. A proposition eliminates possible worlds incompatible with that proposition, for example p eliminates all worlds where p is not the case. In inquisitive semantics, the notion of a proposition is enriched to include one or more possibilities. Possibilities are potentially overlapping sets of worlds that make that proposition true. For example, a purely informative utterance p forms a single possibility that contains the worlds $< pq >$ and $< p\overline{q} >$ as shown in the figure below.



(1) $\lceil p \rceil$

Inquisitiveness

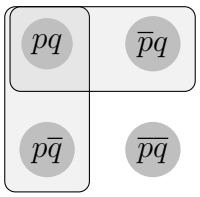
Any proposition that includes two or more possibilities is inquisitive, which allows us to formulate the question "Is p the case?" as $\lceil p \vee \overline{p} \rceil$ which forms two non-overlapping possibilities, one of which is a set of worlds where p is the case and another where $\overline{p}$ is the case.



(2) $\lceil p \vee \overline{p} \rceil$

Disjunction

Classically disjunction generates one possibility, which only eliminates the world $< \overline{pq} >$. In DRIS each disjunct generates one possibility, which makes disjunctions a source of inquisitiveness. But a disjunction itself is not just an inquisitive utterance, but a hybrid as it consists of two possibilities but also eliminates the world $< \overline{pq} >$. Thus, disjunction in DRIS provides both information and requests a response from the hearer.



(3) $\lceil p \vee q \rceil$

Negation

The contribution of negation in DRIS is to give the information that the speaker's state rejects the utterance. Unlike standard accounts, DRIS explicitly defines the sets of possibilities that constitute the rejection of utterances. In standard inquisitive semantics, negation is defined as the complement of an utterance, which necessarily makes all negations assertions. To allow for inquisitiveness in negations, DRIS defines negation as the counter-possibililities that make the negation of an utterance true. Thus, $\lceil \overline{\varphi} \rceil$ becomes $\lfloor \varphi \rfloor$, which is also a set of possibilities. Such an approach also allows us to have a more sophisticated view on the negation of conditionals and conjunctions.

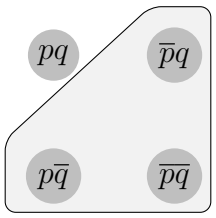
Conditionals

The definition of conditionals in DRIS is connexive, which is to say that $\lceil p \to q \rceil$ forms a single possibility with all worlds except those in which both p and $\overline{q}$ is the case and the negation of a conditional $\lfloor p \to q \rfloor$ forms one possibility which includes all worlds except those in which both p and q is the case. Such a definition of the negation of conditionals is motivated by both natural language and logical puzzles for material implication.

Grice provides the following example regarding the negation of conditionals.

(6) "If god exists, we are free to do what we want."

The unfortunate consequence of a standard definition of conditionals is that if someone disagrees with (6) then they eliminate all possible worlds except for those where god exists and we are not free to do what we want. Regardless of one's beliefs regarding the existence of god, very few would say that the negation of (6) proves the existence of god, or that god's existence is required to negate the utterance. A more intuitive formulation of the negative response could be paraphrased as "If god exists, we are not free to do what we want."



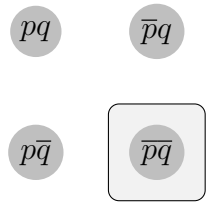
(4) $\lfloor p \to q \rfloor$

Another problem comes from [Read, 1993]. $(p \to q) \vee (q \to p)$ is not intuitively a tautology, but it cannot fail to hold true under classical definitions for disjunction and conditionals. This is due to the fact that if the negation of a conditional in the first disjunct is taken classically to eliminate all worlds except for $< p\overline{q} >$, then the second disjunct cannot fail to be true and *vice versa*. DRIS does not share the problem as the negation of either conditional $p \to q$ or $q \to p$ comes out as on the figure above. Both negations eliminate the world $< pq >$ and allow for the other three worlds.

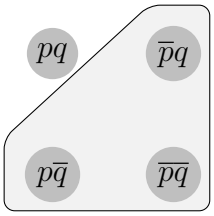
Note that $\lceil \varphi \to \psi \rceil$ can be inquisitive if the consequent is inquisitive and $\lfloor \varphi \to \psi \rfloor$ can be inquisitive if the antecedent is inquisitive.

Conjunction

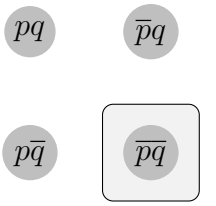
In standard inquisitive semantics, there exists an asymmetry between disjunction and conjunction. While $\overline{p \vee q} \models \overline{p} \wedge \overline{q}$, $\overline{p \wedge q} \not\models \overline{p} \vee \overline{q}$. This is because the negation of conjunction is taken to be a complement, which is a single possibility that cannot be contained in the two possibilities for disjunction.



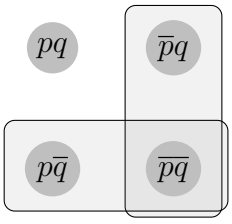
(5) $\overline{p \vee q}$



(6) $\overline{p \wedge q}$

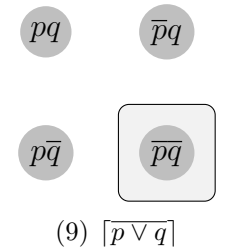


(7) $\overline{p} \wedge \overline{q}$

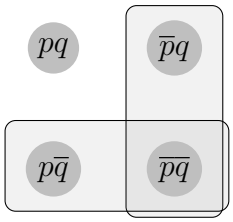


(8) $\overline{p} \vee \overline{q}$

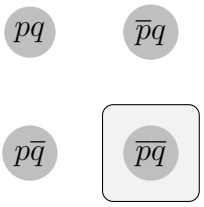
In radical inquisitive semantics, the negation of conjunction is inquisitive, which restores the symmetry.



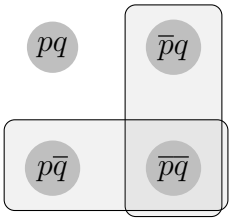
(9) $\lceil \overline{p \vee q} \rceil$



(10) $\lceil \overline{p \wedge q} \rceil$



(11) $\lceil \overline{p} \wedge \overline{q} \rceil$



(12) $\lceil \overline{p} \vee \overline{q} \rceil$

Further evidence for the inquisitiveness of the negation of conjunction comes from the fact that after such an utterance, one can reasonably ask: "Why?", expecting the interlocutor to specify which conjunct was unacceptable.

Entailment

The idea behind drawing valid conclusions through inference is that the conclusion provides less information than the preceding step. This should mean that if one did not have a problem with the preceding step, then the following step cannot be problematic either. This is true in a classical setting and one can generalize that $\varphi \models \psi$ iff $\neg\psi \models \neg\varphi$. Unfortunately, standard inquisitive entailment does not guarantee the validity of inferences in a radical setting where negation is not always a complement. Standard inquisitive entailment is as follows.

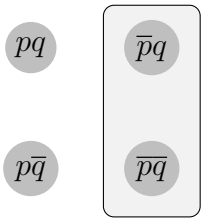
(7) $\varphi \models \psi$ iff $\forall \alpha \in \lceil \varphi \rceil : \exists \beta \in \lceil \psi \rceil : \alpha \subseteq \beta$

[Lewis and Langford, 2000, Originally 1932] give a test for entailment. According to them p entails q if it's logically impossible that $p \wedge \overline{q}$. Standard inquisitive entailment says that $\lceil \overline{p} \vee q \rceil$ entails $\lceil p \to q \rceil$, but this does not pass the test as the intersection of $\lceil \overline{p} \vee q \rceil$ and $\lceil \overline{p \to q} \rceil$ is $\lceil \overline{p} \rceil$ rather than a contradiction ie. emptyset. This suggests that radical inquisitive semantics requires a new definition.

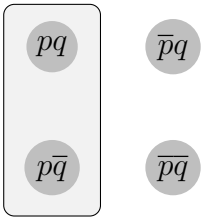
Definition 5. Radical Entailment
$\varphi \models \psi$ iff $\forall \alpha \in \lceil \varphi \rceil : \exists \beta \in \lceil \psi \rceil : \alpha \subseteq \beta$ and $\lfloor \psi \rfloor \subseteq \lfloor \varphi \rfloor$

What radical entailment says is that when φ entails ψ, then not only must every possibility in φ be contained in a possibility for ψ, the counter-possibilities in ψ must be contained in the counter-possibilities for φ.

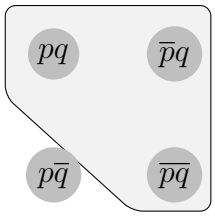
A radical definition of entailment restricts the available inferences in a way that makes a violation-based deontic logic more plausible. It can deal with a number of issues for previous semantic models, which will be expounded in a following section.



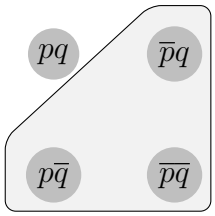
(13) $\lceil \overline{p} \rceil$



(14) $\lfloor \overline{p} \rfloor$



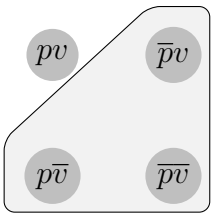
(15) $\lceil p \rightarrow q \rceil$



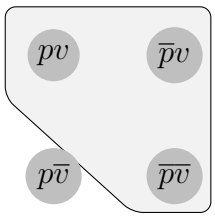
(16) $\lfloor p \rightarrow q \rfloor$

Permission Utterances

The effect of uttering $\lceil \Diamond p \rceil$ provides the information that doing p will not lead to a violation. So the world $< pv >$ is eliminated. The utterance of a permission utterance does not prejudice whether p or $\overline{p}$ is the case, nor does the negation $\lfloor \Diamond p \rfloor$. The possibility for $\lceil \Diamond p \rceil$ is a set of worlds that includes for example the world $< \overline{pv} >$ in which p is not the case. This accounts for the intuition that permission sentences do not require one to in fact perform the act that is permitted. Furthermore, the possibility includes the world $< \overline{p}v >$, where a violation does occur, but p is not the case. This world allows for a more fine-grained analysis of interaction between different permissions and prohibitions, as a permission for one thing, in this case $\lceil \Diamond p \rceil$ does not guarantee that a violation may not occur when another thing, for example r, is the case.



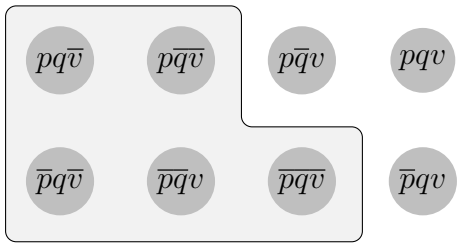
(17) $\lceil \Diamond p \rceil$



(18) $\lfloor \Diamond p \rfloor$

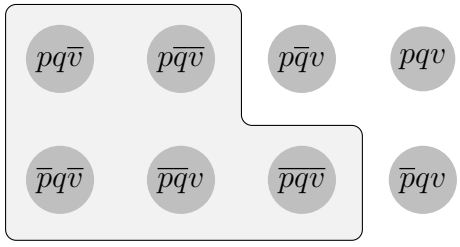
For this account to provide a solution to the free choice puzzle, we must interpret "may" as taking scope over disjunction. And, indeed, this interpretation follows from general observations

regarding disjunction and scope. Following [Eckardt, 2007, pp. 9-10] we argue that in case of ambiguities, one chooses the strongest of the alternatives. In this paper, strongest is understood classically as most eliminative, while the standard measure for strength in inquisitive semantics is homogeneity [Groenendijk and Roelofsen, 2009a, p. 23], which states that a sentence ought to be more eliminative and less inquisitive to be stronger. As can be seen by comparing figures (19) and (23), "may" taking scope over disjunction provides the stronger reading. In this formulation, a disjunctive permission sentence eliminates three worlds: $< p\overline{q}v >, < \overline{p}qv >$ and $< pqv >$. The result is a single possibility that includes the remaining worlds.

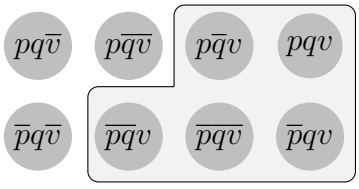


(19) "A country may nominate a state funded research center or a private laboratory."

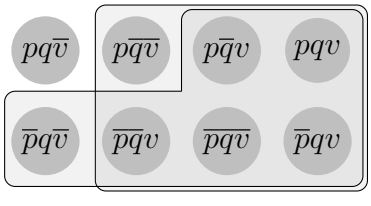
This appears to be in line with our intuitions regarding permission being granted for both disjuncts. Note that contra Barker, the semantics of permission and conditionals differs to provide an entailment based account for the boolean negation of free choice utterances.



(20) $\lceil p \vee q \rightarrow \overline{v} \rceil / \lceil \Diamond (p \vee q) \rceil$



(21) $\lfloor \Diamond (p \vee q) \rfloor$



(22) $\lfloor (p \vee q) \rightarrow \overline{v} \rfloor$

The natural language examples that correspond to the above figures are as follows.

(8) (a) A country may establish a research center or a laboratory/If you drink alcohol or smoke at school, you will not break the law.

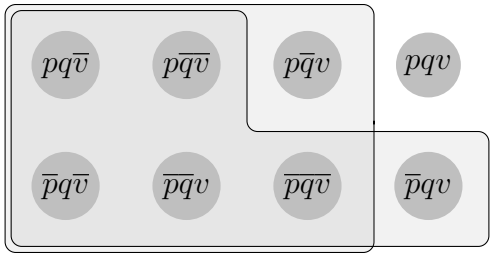
(b) A country may not establish a research center or a laboratory.

(c) It's not the case that if you drink alcohol or smoke at school, you will not break the law.

[Simons, 2005a, Zimmermann, 2000] draw our attention to the possibility of appending a disjunctive permission sentence with "... but I do not know which." such that the free choice effect gives way to an ignorance reading.

(9) A country may establish a research center or a laboratory, but I do not know which.

The entailment relations of these examples revert back to standard disjunction. We argue that this effect arises from the additional information blocking the modality from taking strongest scope - scoping over the disjunction. The result is a translation of (9) as a wide scope reading in which disjunction takes scope over "may" such that "may" distributes into the disjuncts. As disjunction is a hybrid between inquisitive and informative utterances, a wide scope disjunctive permission utterance raises an issue for the speaker to solve, modeled as two possibilities as shown in the graph below.



(23) "A country may establish a research center or a laboratory but I don't know which."

DRIS still includes classical negation that takes the complement of a proposition. This allows us to model yet another effect. The following utterance is appended with "... and, also, I do not care."

(10) A country may establish a research center or a laboratory, but I do not know which, and, also, I do not care.

We model "I do not care." through double classical negation: $\neg\neg\lceil\Diamond\varphi \vee \Diamond\psi\rceil$ (known as non-inquisitive closure in Groenendijk and Roelofsen [2009a, p. 4]), which results in the same possible worlds as (9), but it consists of only one possibility. This correctly does not guarantee that doing p or q does not result in a violation. Note, that the non-inquisitive closure only has an effect on inquisitive utterances, and "I do not care" is superfluous when added to the free choice examples.

[Simons, 2005a] draws attention to the fact that the following utterance can also receive a free choice reading.

(11) A country may establish a research center or a country may establish a laboratory.

The conditions for such a reading to arise have not been in the literature, and here it is assumed that the permissive "may" in either disjunct follows the general scope principle of taking strongest scope, which means that both take scope over the entire disjunction, effectively having the same effect, which yields the usual free choice reading depicted in (19).

The consequence of the above semantics and definition for entailment is that $\lceil \Diamond(p \vee q) \rceil$ no longer entails $\lceil \Diamond p \wedge \Diamond q \rceil$ or $\lceil \Diamond p \rceil$. This result seems to be intuitively correct as we can see if we study the examples more closely.

(12) (a) A country may establish a research center or a laboratory.

(b) A country may establish a research center and a country may establish a laboratory.

On the positive account, (12 a) and (12 b) do not seem to meaningfully differ, and the reading of (12 b) where a country may only establish a research center when it also establishes a laboratory is dispreferred.

(13) (a) A country may not establish a research center or a laboratory.

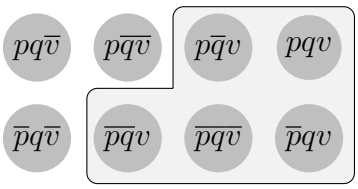
(b) A country may not establish a research center and a country may not establish a laboratory.

(c) It's not the case that a country may establish a research center and that a country may establish a laboratory.

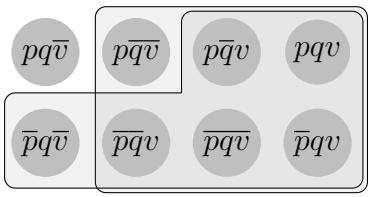
On the negative account, though, there is a difference on the level of the surface structure and meaning. In terms of the surface structure, the most natural location for negation is between "may" and the disjunction as in (13 a), but a negation between "may" and the conjunction is difficult. One would either explicitly negate each conjunct (13 b) or negate the conjunction (13 c). Negating the conjunction does not give a "neither" reading as with permission, and one can naturally ask which of the conjuncts is false. As negating either conjunct is doing something in addition to the default negation of a conjunction[2], entailment only takes into account default negation and is thus blocked between $\lceil \Diamond(p \vee q) \rceil \not\models \lceil \Diamond p \wedge \Diamond q \rceil$ because the preceding statement is more difficult to negate than the following.

This also suggests why a free choice permission utterance intuitively does not guarantee that one is permitted to do both p and q. While on the positive account they eliminate the same worlds, their rejection conditions differ in that to reject the conjunction, one breaks the connection between the disjuncts, allowing one disjunct to be negated while the other is not. This is possible through doing something additional such as sluicing, but not the case by default.

[2]The reading is the same as providing a neither-nor negation of $(p \vee q) \rightarrow \overline{v}$.



(24) $\lfloor\Diamond(p \vee q)\rfloor$



(25) $\lfloor\Diamond p \wedge \Diamond q\rfloor$

It necessarily follows from $\lceil\Diamond(p \vee q)\rceil \not\models \lceil\Diamond p \wedge \Diamond q\rceil$ that $\lceil\Diamond(p \vee q)\rceil \not\models \lceil\Diamond p\rceil$ ie. that a free choice permission utterance does not entail its own disjuncts. At first glance this seems problematic because a free choice permission utterance $\lceil\Diamond(p \vee q)\rceil$ gives permission to do for example p. But the reason for uttering a disjunction instead of a conjunction is because the former is slightly more restrictive. $\lceil\Diamond p\rceil$ provides less information as it is indifferent as to whether $\lceil\Diamond q\rceil$ or $\lfloor\Diamond q\rfloor$ is the case. As a free choice utterance does not guarantee that doing both p and q is not a violation, one cannot unconditionally guarantee that after $\lceil\Diamond(p \vee q)\rceil$ has been uttered $\lceil\Diamond p\rceil$ holds as well as that depends on the status of $\lceil\Diamond q\rceil$. Fortunately, radical entailment blocks such inferences.

Counterarguments Countered

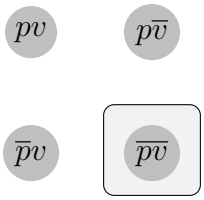
The classic problem with violation based deontic logics is that from the assumption p and the definition of obligation as $\overline{p} \rightarrow v$ one can derive the validity of $p \rightarrow \Box p$ which is an obviously false prediction, known as the naturalistic fallacy. It isn't valid to derive from the fact that something is the case that it is also obligatory. The manner in which [Anderson, 1967] derived this is the following.

(14)
1. $\Box p := \overline{p} \rightarrow v$
2. p
3. $\overline{\overline{p}}$
4. $\overline{\overline{p}} \vee v$
5. $\overline{p} \rightarrow v$
6. $p \rightarrow \Box p$

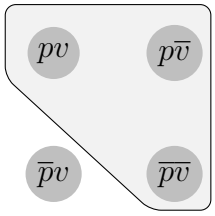
Anderson's derivation poses a technical challenge, but it is not intuitively plausible that this counterargument will hold in natural language semantics. Note that $\overline{p} \rightarrow v$ and $\Box p$ can be assumed to have the same truth conditions, although not all implications with a violation as the consequent and prohibition utterances are equivalent. The fact that $\overline{p} \vee v$ and $p \rightarrow v$ are informatively equivalent, but unavailable for use as substitutes in natural language was already noted by [Grice, 1989a, p. 67].[3] The steps that are most problematic are those from 3 to 4 and 4 to 5. When one knows p or $\overline{\overline{p}}$, it is dubious to assume that disjunctive addition does not create problems. It raises threefold issues. Firstly, one adds inquisitiveness, which previously wasn't present. Secondly, in a system such as [Ciardelli et al., 2009], one would draw attention to a new possibility, which needs

[3]Grice also added "not both p and not v" as an informatively equivalent substitute that people are unhappy to use.

to be justified. And, thirdly, one should be able to attest to the relevance of the added proposition. Unfortunately, challenging any of these three assumptions would require the addition of a great deal to this framework, so noting the limitations for this paper, we decided to concentrate on the move from step 4 to 5.



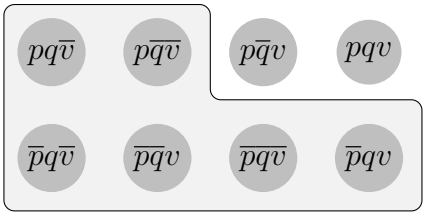
(26) $\lfloor \overline{\overline{p}} \vee v \rfloor$



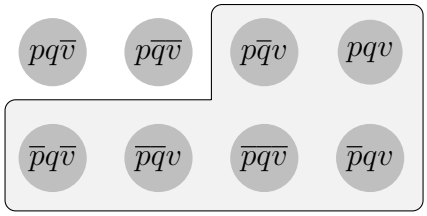
(27) $\lfloor \overline{p} \rightarrow v \rfloor$

As can be seen on figures (26) and (27), to reject 4, one must eliminate two more worlds than to reject 5. It follows that a person that did not have enough information to reject 4 might have enough information to reject 5. Thus, radical entailment blocks the inference of $\lceil \overline{p} \rightarrow v \rceil$ from $\lceil \overline{\overline{p}} \vee q \rceil$.

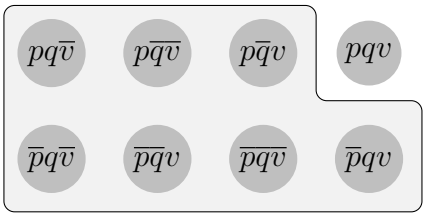
The radical definition of entailment also deals with the problem of strengthening the antecedent. If it is permitted to eat an apple, then intuitively it should not be the case that both eating an apple and killing a postman should still lead to a non-violation world.[4] As one can see in the following figures, this inference is blocked by radical entailment.



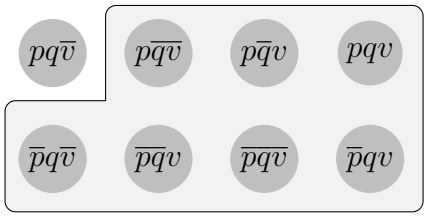
(28) $\lceil \Diamond p \rceil$



(30) $\lfloor \Diamond p \rfloor$



(29) $\lceil \Diamond (p \wedge q) \rceil$



(31) $\lfloor \Diamond (p \wedge q) \rfloor$

As the comparison of (31) and (30) demonstrates, the counter-possibility $\lfloor \Diamond (p \wedge q) \rfloor$ is not contained in the counter-possibility $\lfloor \Diamond p \rfloor$, which means that according to radical entailment the strengthened antecedent is not entailed by the original permission utterance.

[4] In the real world, eating an apple and murder would not be governed by the same violation, but to strengthen the argument, let us assume it is the same violation.

It also no longer holds that $\lfloor p \rfloor \models \lceil p \rightarrow q \rceil$ or in other worlds that a false antecedent makes every statement true. This means that the fact that p did not occur, does not necessitate that there exists a law that makes doing p a violation.

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