

Analogical Reasoning: A Core of Cognition

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Analogies have always been considered a central part of human intelligence and cognition. This survey offers an overview of analogical reasoning and its applications, showing that analogies are an important element of various cognitive abilities like memory access, adaptation, learning, reasoning, and creativity. Therefore, analogies can provide a basis for integrated large-scale cognitive systems.

1 Introduction

Analogy research has a long history and was subject of investigation on various levels: Ancient Greek philosophy examined analogy as a proportional relation between objects. In belletristic texts, metaphors use analogy implicitly to describe objects in a figurative way. In natural sciences, analogy is an indispensable tool for scientific progress [18].

While classical research on analogy mainly deals with higher-level intelligent behavior, the focus in cognitive science has shifted to the examination of more basic cognitive abilities, relevant for agents interacting with their environment. Many of these abilities seem to rely on analogy as well and thereby give an explanation for the analogical capacity on the higher-level. Therefore, it is not astonishing that analogy plays a central role in cognitive science research.

2 What is Analogy?

Analogy making is a highly sophisticated cognitive process in which two conceptualizations - a source and a target - are analyzed for common structural patterns [6]. In analogies, source and target are typically of different domains (for metaphors this is even essential). The purpose of analogies is to adapt knowledge available about the source conceptualization such that it can be applied to the target in a way that new analogous inferences can be drawn. Analogy making requires intelligence since analogous patterns and transfers often are not obvious and depend on a certain conceptualization of the domains.

Metaphors and analogies occur in a large variety of domains, as well as in quite different forms. In order to classify certain aspects and properties of analogies, three types of analogies are often distinguished¹ [14, 22]: First, proportional analogies have the general form $(A : B) :: (C : X)$. Proportional analogies can be *in domain* which means that A, B, C are expressions from the same domain. This type is characteristic for intelligence test where the subjects have to continue a sequence of geometric figures [4] or numbers and strings [12]. Proportional analogies can also be *cross domain*. In this case, A and B establish a relation in the source domain which should be applied to a concept C of the target domain to get the result X in the target domain.

¹We do not claim that the following classification of analogies is complete, nor that it is the only possible one. Rather such a classification can be useful to specify different properties of analogies.

Proportional analogies have a nice counterpart in metaphoric expressions. For example, a statement like *Gills are the lungs of fish* can be analyzed as the solution to the following analogy: $mammals : lungs :: fish : X$. Here the word *gills* refers to whatever the analogical relation constructs for X . What is mentioned in the surface structure of the metaphoric expression is typically a cross domain role relation meaning X plays the role of *lungs* to fish, while the reference to the counterpart of *fish* (*mammals*) is assumed to be derivable by background knowledge.

A second type of analogies are predictive analogies. They describe a new domain (target) not only by specifying structural commonalities with a given domain (source), but also by transferring information and explanations from the source to the target [5, 13]. Besides the classical domain of metaphoric expressions, complex conceptualizations of correlations between seemingly different physical domains are good examples for such analogies. They relate physical domains which are hardly accessible by our direct experience to domains which have perceivable properties. The importance of analogies becomes evident in cases where such working analogies are not available. Why is relativity theory or quantum mechanics so hard to conceive by humans? The main reason is, because the analogical transfer to a domain which is accessible to our direct experience essentially fails. There are many other examples in modern physics where no good analogies are available or strange metaphors are used which do not explain very much: consider for example the wave-particle dualism, exchange forces (based on photons, gravitons etc.), or string theory.

The third type of analogy, analogical problem solving, can be used to solve a problem by transferring a solution from a well-known domain to an unknown domain. Case-based reasoning [16] formalizes approaches of this type. Other examples are the famous tumor-fortress problem [8] or the usage of program code for developing new programs by abstraction [23].

Another aspect connected to analogies is the reconceptualization of known domains (called *rendition*). This may occur if an obvious analogy fails. A classical example is the conceptualization of a paintbrush as a tool to transfer paint to a canvas by pumping paint in contrast to smearing it [14]. In general, verifying hypotheses about allegedly known (real world) situation (source) by experiments in a controlled lab environment (target) uses an analogical relation between both domains. The outcome of such an experiment may force us to reconceptualize the source domain.

It is often mentioned that metaphors are strongly connected

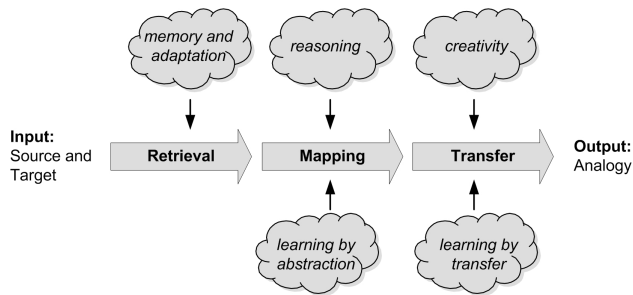


Figure 1: Analogies in cognitive processes.

to analogies [7, 14]. Depending on the context one and the same metaphor can be proportional or predictive. Assume for example a situation in which a teacher is lecturing students on elementary atom physics by giving the following metaphor describing the Rutherford analogy:

- (1) Electrons are the planets of the atom.

(1) can be interpreted as a predictive analogy: The students learn a new conceptualization of the atom. On the other hand, for a scientist (1) can be simply interpreted as a (historically important) proportional analogy.

The process of analogy making can be subdivided into several, interrelated tasks. Typically, the following three subtasks can be identified: *retrieval*, *mapping*, and *transfer*.

At the beginning, when exposed to a new situation (the target), a source domain has to be identified to which that situation can be related. Some retrieval technique has to be applied to search the memory for items which seem like candidates for an analogy. In certain settings, such as intelligence tests or teaching situations, the source domain may be given explicitly. Some models for analogy making even view this as the standard case and do not provide special means of retrieval.

The mapping phase aims to establish the analogical relation between these two domains, i.e. the alignment of structures from both domains. In general, there are many possible mappings and which one is appropriate depends on the context and the goal of the analogy. Two problems are typically discussed associated with the mapping step. The *relevance problem* consists of identifying the parts of the domains that are relevant within the context of the analogy and therefore shall enter the analogical relation. The *representation problem* is concerned with the difficulties in mapping caused by differently structured representations of the domains. While it may seem plausible that two domains are represented in an isomorphic way when they are especially prepared for the analogy, this seems unlikely in general. Therefore the mapping may guide a restructuring of one or both domains and a good deal of the explanatory and creative power of analogies can actually be ascribed to that process of re-representation.

During the transfer phase the analogical relation is used to translate information between the two domains. Normally knowledge is transferred from the source to the target domain and is used there to introduce new concepts or structures, give new explanations to phenomena, or solve given problems. This new knowledge is in no way logically justified and should merely be seen as a hypothesis, but when used carefully, it can be the

source of valuable inspiration.

In some cases, these three phases are supplemented by additional steps, such as evaluation of the transferred knowledge, or the induction of generalized rules, depending on the model applied and the context in which analogy making is placed.

3 Cognitive Abilities in Analogies

Analogies are paramount for understanding many cognitive phenomena. Figure 1 illustrates the process of analogy making and relates cognitive abilities to the different stages. In the following, we describe in detail each of these abilities.

3.1 Memory and Adaptation

The ability to store information and recall it on given occasions is crucial to an appropriate use of memory. As every new situation includes a lot of details making it unique and different from previous experiences, a retrieval process making memories usable for that situation heavily relies on analogy. Irrelevant details must be blinded out, old patterns have to be mapped to new domains. So the recall of information from memory is a sophisticated process with close resemblance to the phases of retrieval, mapping and transfer described above.

Kokinov [15] proposes a cognitive architecture in which memory plays the central role, to address cognitive processes like analogical reasoning and the estimation of semantic similarity. The system consists of a network, the DUAL memory, representing the system's world knowledge. The nodes of the network, called micro-agents, have an inner structure and are in a state affecting their behavior in cognitive processes. Such processes are triggered by activation potential and marker passing along the connections between the micro-agents. An initial activation pattern is determined by context and then propagated according to the characters of the individual agents and their interconnections. The activation pattern represents the contents of the short-term memory, while the whole micro-agent network corresponds to long-term memory.

Based on this memory architecture, different types of retrieval and reasoning can be performed. For example, the framework has been demonstrated to be capable of analogical problem solving: given the task to heat water in a wooden vessel in a forrest with only matches and a pocket knife the system proposed, analogous to an immersion heater, to warm up the knife and then put it into the water.

A totally different approach to memory are Vector Symbolic Architectures (VSAs) [19]. This approach uses high-dimensional vectors to store information by simple mathematical operations like addition and multiplication. Storage and retrieval can be carried out via the same set of operators which can also be used to realize some kind of analogical access. From a vector that holds the information *Paris is the capital of France* and *Stockholm is the capital of Sweden*, questions like *What is the relation of Paris and France?* and *What is the capital of Sweden?* can be answered. But it is also possible to retrieve statements not explicitly stored in the memory, such as the solution to the analogy task *Paris is to France like X is to Sweden*.

3.2 Reasoning

Human cognitive reasoning capacities include the classical inference mechanisms: Deduction infers knowledge given implicitly by known facts and rules: if the premises evaluate true, deduction infers according to the rule that the conclusion is true as well. Inductive reasoning creates knowledge via generalization over a large set of cases which share a common fact or rule and induces a general rule. Abduction is reasoning from an effect to its cause: from a known general rule and a true conclusion it is hypothesized that the premise is true as well.

Beside these classical reasoning mechanisms, human reasoning capacities include analogical reasoning, too. Moreover, analogical reasoning can imitate the classical forms of reasoning: if an analogy completes a causal relation on the target side by a missing premise it resembles an abductive inference and if it transfers the conclusion it resembles a deductive inference.

While a deductive inference is truth-preserving, inductive, abductive and analogical inferences are not. But the likelihood of inductive inferences increases with the number of valid cases. Therefore the set of cases must be sufficiently large to draw sound conclusions. Analogical reasoning, however, requires only two cases: the likelihood of analogical inferences increases with the number of analogical matches between the source and the target [25].

We distinguish two types of analogical inferences: Either it comprises only the transfer of a structural relationship between two elements. Or analogical transfer refers to a general rule. Applying the rule from the source domain, new inferences can be drawn in the target domain. However, in both cases the analogical inference is based on the uncertain analogical mapping which must not necessarily be true.

3.3 Learning by Transfer

In artificial intelligence, a large number of different learning theories were proposed. Two major types of learning theories can be distinguished: *Lazy learning algorithms* store each example explicitly in a database without identifying abstract features. Instance-based learning [1], case-based learning [16], or memory-based learning [24] are representatives of this learning type. *Eager learning algorithms* minimize the storage load by an abstraction process. The identification of important (common) features of the examples is a necessary prerequisite for this type of learning algorithm. Examples for techniques of learning are decision-tree learning [21], inductive learning [17], or connectionist-style learning [2].

Both classes of learning strategies are based on a common idea: As input data a (more or less) large sample of examples is needed to guarantee a successful learning procedure. Contrary to these approaches, cognitive learning seems to be based on a significantly less amount of training data and occurs often in the form of implicit learning. We propose to explain these discrepancies by analogical learning.

In the case of analogical learning a conceptualization of the source domain is sufficient to generate knowledge about the target domain. Moreover, a projection of attributes and relations from the source to the target can productively introduce new concepts on the target domain. As a result cognitive agents can learn a new conceptualization of the target domain without perceiving a huge number of examples.

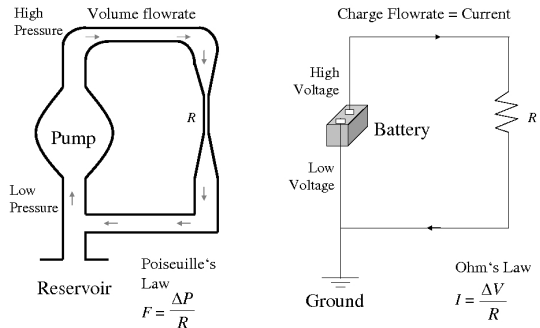


Figure 2: The analogy between a water pipe system and an electric circuit in a diagrammatic representation. The Figure contains more information than necessary for an interpretation of the metaphorical description.

A prototypical situation in which analogical learning plays an important role are teaching situations. When the teacher explains an electric circuit by analogy with a water-flow system (Figure 2), the student needs to establish an analogical relation between the involved domains.² Only if the student successfully transfers the flow of water to the target domain she will reach the conclusion that current is flowing in the electric circuit.

A famous approach for such types of transfers is the *structure mapping theory* (SMT) where two graphs are used to represent models of the source and target domain [5]. An alignment process searches a maximal match of subgraphs. The algorithm returns an analogical relation between the two domains. Learning in SMT is realized by transferring relevant parts of the source graph which have not been aligned [6]. The alignment and transfer process is guided by principles ensuring consistency of the hierarchical relational structure.

3.4 Learning by Abstraction

Whereas a transfer of information from the source to the target yields a new conceptualization of the target domain, learning in the classical sense is concerned with finding hypotheses that generalize the training examples to unseen test examples. This aspect is covered by learning by abstraction: Instead of a plain mapping, the following two-layered procedure is considered. First, identify the common structure of source and target by a generalization. Second, calculate the mapping of objects of the source to objects of the target via the common structure. Particularly with respect to certain application domains of analogies like the string domain this abstraction process seems to be crucial. In Hofstadter's analogy model Copycat [12], string sequences like *ababab* need to be represented at a more abstract level as $3 * (ab)$ to recognize the analogical relation to strings like *ccc*.

A formal theory for computing analogies via an abstraction process is proposed in [11]: Heuristic-Driven Theory Projection (HDTP) is an algorithmic approach to calculate a common generalization of a source and a target. The underlying mathematical model for such generalizations is anti-unification [20]. Constraints are given by the problem structure or by the functional role the objects play in a given structure.

²c.f. <http://hyperphysics.phy-astr.gsu.edu/hphys.html>

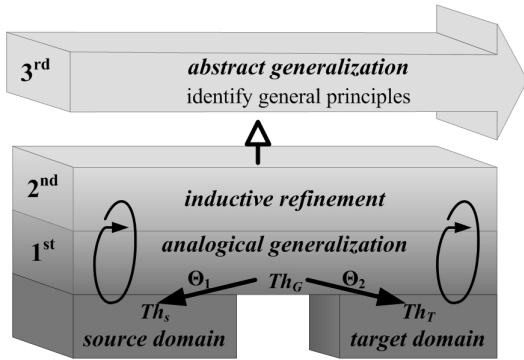


Figure 3: A graphical representation of learning stages.

In HDTP’s logical framework, source, target, and generalization are given by theories Th_S , Th_T , and Th_G respectively. Comparing $attracts(sun,planet)$ from Th_S to $attracts(nucleus,electron)$ to Th_T in a logical representation of the Rutherford analogy (section 2), this procedure would result in the generalization $attracts(central_body,orbiter)$ in Th_G , where $central_body$ and $orbiter$ are variables. (1st level of Figure 3). By calculating the abstraction, the mapping $\{central_body \mapsto sun/nucleus, orbiter \mapsto planet/electron\}$ is obtained which represents two substitutions: Θ_1 from Th_G to Th_S and Θ_2 from Th_G to Th_T . The generalized theory Th_G comprises the common structures of source and target: in the Rutherford analogy the generalization of the solar system and the atom can be conceptualized as a central force system.

In HDTP, learning occurs as a side-effect of the modeling and can be considered as being implicit: the generalization process yields new conceptualizations of the target domain at an abstract level for no additional costs. In a uniform way, it provides a structural description of the underlying domains in terms of a generalized theory.

Analogical learning in HDTP does not end with a successful establishment of an analogical relation, but learning continues in stages. After the most specific generalization is established (1st level), the inductive refinement (2nd level) adjusts the parameters in order to find a new (and finer) conceptualization of source an target. The 3rd level aims to identify general principles that can be applied in a variety of domains.

In plain structure mapping as well as in mapping via abstraction, the greatest common sub-structure of source and target must be identified. But in mapping via abstraction, the common structure is not forgotten but represented explicitly. Instead of mapping one object directly onto another, it is mapped via the role it is playing in the common structure.

3.5 Creativity

Creativity as defined in [3, p. 205] is “the production of an idea, action, or object that is new and valued”. Analogies can provide a means for creativity, as they can introduce new concepts into a domain via analogical transfer. The principles guiding the transfer ensure that the concepts are relevant in order to understand the target domain. Furthermore analogies can induce new insights about a domain by reconceptualizations.

From the very beginnings, artificial intelligence struggles hard to develop models for creativity. Nevertheless these

endeavors were not very successful so far. Perhaps analogical reasoning can provide a way out of this dead end.³ Examples of analogy related creativity can be seen in natural language:

- (2) He reached the maximal height of his career.
- (3) Gills are the lungs of fish.
- (4) My brain is a little foggy.
- (5) Juliet is the sun.

These metaphors govern the creation of new concepts to a different extent. *Height of career* in (2) seems to be a conventional metaphor, i.e. the constituent is lexicalized and therefore an idiom. In (3) the new concept *gills* is strongly determined by the source domain. In (4) the meaning of the adjective *foggy* has to be adapted to be applicable to brain states. Shakespeare’s metaphor in (5) is at a very abstract level and it remains relatively vague which properties are transferred.

Another domain for which creativity in analogies is an important aspect are examples from the physical domain like the former mentioned water-current analogy.⁴ Figure 2 depicts the situation between a water-flow circuit and an electric circuit. Although there is no possibility to observe current flowing in an electric circuit directly, humans are able to establish such an abstract concept in analogy to water flowing in a water pipe system.

How can analogical reasoning explain these creative abilities? An answer can be provided by combining the transfer of facts and rules from the source to the target (section 3.2) with abstraction and generalization (section 3.4). In (6), current is explicitly associated with water, but the properties of current are not specified. In fact, even if a student has no idea which properties should be ascribed to current (for example, in a learning scenario), (6) could be the cue to get the *flowing* conceptualization of current by transferring properties of water from the source to the target. This transfer enables the learner to create properties of current like *flowing-in-circuit* or the necessity of a pressure source, provided the modeling and the background knowledge is fine-grained enough.

In particular, analogies in qualitative reasoning applied to the physics domain are good examples for creative concept formations: a famous and extensively discussed example is the creation of the concept *heat* in a water-flow vs. heat-flow scenario [22, 9]. In this scenario, the abstract concept *heat-flow* needs to be creatively generated by the analogy, although heat is neither observable nor measurable. Projecting the situation in a water-flow system to the situation in which the temperature of a cup of coffee is decreasing and the temperature of a metallic cube is increasing (both are connected by a metal bar), the successful establishment of an analogy introduces a concept *heat-flow*, explaining the changes in the setting.

4 Conclusions and Future Directions

In this article, we argued that analogical reasoning is a crucial issue for cognition. Classical cognitive abilities of human agents such as memory capacities, learning by transfer, learning by abstraction, forms of reasoning, and creativity can be considered

³Examples of creative aspects of analogies can be found in [14].

⁴A precise modeling of this analogy can be found in [10].

as necessarily interconnected with and in an important respect governed by the capacity to establish analogical relations. In a certain sense these capacities (typically considered to be distinct) can be explained by analogies in a rather unified manner.

Clearly the proposed modeling of a variety of cognitive phenomena can be interpreted as a reductionist position of cognition where several seemingly incoherent aspects of cognition can be reduced to one general principle underlying the whole system. Although we clearly do not claim that cognition as a whole is nothing else but analogy making, the possibility of a reduction to a main principle allows to speculate about the chances to build integrated large-scale cognitive systems: the vast amount of information, the different computing paradigms in AI for modeling particular aspects of cognition, and the problem of modeling dynamic changes of a cognitive system make it difficult to come up with a large system. Analogical reasoning makes a step towards the very possibility of this endeavor.

Acknowledgement

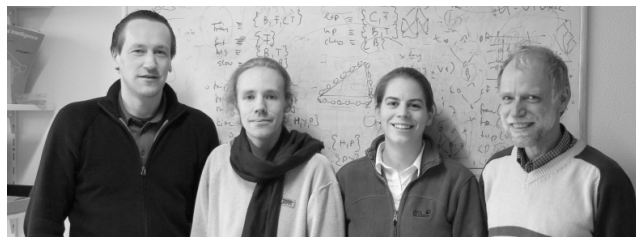
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The analogy group at the Institute for Cognitive Sciences in Osnabrück (from left to right: Kai-Uwe Kühnberger, Ulf Krumnack, Angela Schwering, and Helmar Gust). Analogy has been a topic of interest for the last years and has led to the development of HDTP, a formal model for analogy making.